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THE USE OF SIMULTANEOUS EQUATION MODELS FOR DECISIONS PERTAINING TO THE "BEST" MIX BETWEEN AIRCRAFT, SPARE PARTS, SUPPORT EQUIPMENT, AND SUPPORT PERSONNEL

Chantee Lewis, Capt., U.S.N.

Research Contribution 206

Center for Naval Analyses

Naval Warfare Analysis Group

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2. This research contribution describes and demonstrates the use of a method to determine the mix of aircraft, men, spare parts, and support equipment that will maximize sorties or ready hours within budget constraints. The approach is generally applicable to many Navy resource allocation problems and should be useful to Navy activities with budgetary, program management, and material support responsibilities.

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ABSTRACT

This is a study of the application of production functions to sea-based tactical air resources: aircraft, spare parts, support equipment, and support personnel. The goal is to develop objective criteria for allocating money among these competing demands using sorties or aircraft ready hours as the output.

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PREFACE

This work was first reported in slightly different form to the School of Government and Business Administration at the George Washington University as a doctoral thesis. It is being offered for wider distribution now because of the author's belief that the methods proposed here can have far-reaching effects in deciding how to allocate scarce funds among what might seem like equally important requirements.

Among those whom the author is indebted to for their contributions to this work are:

- Drs. Jerome Bracken, Guy Black, and N. Singpurwalla of George Washington University,
- Dr. Joseph B. Kadane of Carnegie-Mellon
- Mr. Michael R. Ferguson of CNA.

The data upon which this analysis is based was contributed by several military and industrial organizations, particularly the Maintenance Support Office at Mechanicsburg, Pennsylvania.

SECTION I

INTRODUCTION

OBJECTIVES AND PURPOSE

The relationships between aircraft inputs (airplanes, spare parts, men, etc.) and the outputs of sorties or aircraft ready hours can be approximated reasonably well by a production function type model. It is the initial intent of this research effort to construct and evaluate such an aircraft carrier production function. This model will then permit defense managers to have better understanding of the actual input-output process of operating sea-based tactical aircraft. The model will indicate:

(a) Within the normal range of sortie and maintenance policies, the best policies, the best proportion (allocation) of aircraft, men, support equipment and spares to obtain maximum sortie or ready hour capabilities, subject to various levels of the budget.

(b) Within various constraints, the optimum squadron composition at various levels of cost to achieve maximum outputs for an airwing.

(c) When occurring, possible unused resources or the need of additional resources.

(d) Whether the allocation of men, support equipment, and spares is sensitive to moderate changes in labor rates, rates of discount interest, or variations in the expected operational life of the aircraft weapon system.

(e) The relative "costs" between various combinations of sortie and maintenance policies.

SCOPE

This research concentrates on the "industrial" type production function situation where there is a transformation of materials into products (sorties, availability) by a series of energy applications, all at a cost. Given that some level of aircraft carrier-based sea power is desirable, the study examines the various possible allocation alternatives of inputs to obtain a maximum level of output for various budgets or resource constraints (Example: the space limitation aboard ship).

Only recent (1968-1971) military aircraft maintenance data are appropriate for this study. The level of technology has an impact on production functions; because of rapid changes in military technology, historical data are probably not suitable for predicting or building current models. Unfortunately, the military data reporting procedures for aircraft maintenance actions (3M - Maintenance, Material, Management reports) have changed several times in the last few years. After each change there is a period of time in which some field activities report under the new system and some - erroneously - under the old format; the net effect is that the aggregate data is unusable for this type of research. Thus, the refined time period of maintenance observations used for this study is limited to July through December 1968, all of 1969, and May through October 1970; 24 months of observed data in all.

LIMITATIONS

(a) In general, the inputs and outputs must be viewed on an aggregate basis. Due to restrictions in the 3M reporting format, it is not possible to have meaningful breakdowns of cost categories by specific aircraft carrier, specific model of aircraft, categories of labor (direct/indirect), or type of maintenance policy.

(b) Measurement and allocation efforts are concentrated on expensive aircraft components and support equipment, such as engines, landing gear, and electronic test units. Low-cost items that present no storage or support problems are handled only as a general category of logistic material. This will increase the possibility of bias, but sensitivity checks should indicate whether this is a significant limitation.

(c) A limited attempt is made to quantify the exact cost of specific maintenance and sortie policies. Efforts are made to indicate the relative rank or effectiveness between various policies. A relative evaluation of the outputs may be more appropriate than an attempt to measure the outputs on an absolute basis.

(d) The inputs (support equipment, spares, manpower) have a direct investment or wage cost, plus the cost of the unique sea-based space set aside for these inputs. At present the absolute costs of these types of space are not available. However, the relative cost data of shifting one square foot of space from storage, to support, to living spaces, etc., (subject to the upper limits of the aircraft carrier hull) is available and will be used. This reduces the cost bias against alternatives and presents no significant problem when comparing alternatives aboard existing ships. This would be a limitation if the alternatives included new ships and land-based tactical air.

(e) Where sufficient data is not available, judgment by experts is used to augment the limited information. Any assumptions made by the experts are clearly stated.

(f) The study excludes classified or proprietary information. This limitation has little affect on the use of data in its aggregate form and has no affect on the methodology.

ASSUMPTIONS

This analysis assumes that some portion of our tactical air power should be sea-based. Given this, the maximizing of aircraft outputs, subject to the budget and short-term space constraints aboard a ship, is a logical management objective. To achieve this objective, trade-off analysis between types of inputs is required which in return necessitates an implicit tactical aircraft revenue function. In the public/defense sector, acceptable revenue functions are difficult to establish and measure. However, it is assumed that for ongoing aircraft programs the national utility or revenue return for a specific aircraft is at least equal to or as great as the total investment cost of the aircraft.

STATEMENT OF THE PROBLEM

The problem is to develop a methodology which will serve as a management aid to decision makers so that they can arrive at a near optimal allocation of tactical aircraft carrier inputs (airplanes, spare parts, etc.) to achieve specified outputs.

MEASURES OF EFFECTIVENESS (MOE)

There is no agreement on any specific MOE's for tactical aircraft. They range from measurable items, such as effective sorties, ready hours, and loiter time, to such non-quantitative considerations as raising the national level of technology and providing an industrial base for mobilization. For each specific type of aircraft, the dynamic MOE of sorties and the static MOE of ready hours will be used. Before different aircraft can be compared, however, the MOE units will be normalized for payload (bombs), firepower (air-to-air weapons), and the expected probability of mission success.

RESOURCE ALLOCATION OPTIMIZATION MODEL

The objective function of the model, to be established, is to maximize the level of output for a fixed level of budget, subject to the physical and policy limitations of the situation. In mathematical notation, for one output, the basic objective function is to maximize a Cob-Douglas type equation of:

$$U = \alpha_o W_1^{\alpha_1} W_2^{\alpha_2} W_3^{\alpha_3} W_4^{\alpha_4} \epsilon,$$

subject to the various constraints,

where

U = output

α_o = a scaling efficiency or technology change factor

W_j = inputs

α_j = elasticity with respect to the W_j input

ϵ = the degree of random distributions.

SECTION II

INPUTS AND THEIR RANGE OF COST ASSOCIATED WITH PROVIDING THE AIRCRAFT OUTPUTS

This section reviews the airline and military aircraft literature in order to establish a realistic method of estimating inputs and outputs and their cost for this type of industrial situation. Specified inputs and outputs are used later in a production function type analysis. The concepts involved will be general in scope and for simplicity will employ only basic estimating formulas with an indication of the range of items under consideration.

In general, the primary consumable inputs (fuel, oil, tires, etc.) and their cost are directly proportional to the level of standard operations. Although this input is large, the method of estimating it is trivial compared to the complex problems involved in estimating prorated investment effects. In addition, consumable costs do not play a key role in determining potential flight outputs. Particular attention will be given to high-speed modern aircraft total investment effect, logistic support and the implications of various maintenance and operating policies.

AVIATION OUTPUTS AND INPUTS IN GENERAL

No one single measure of output is satisfactory for the commercial airlines. Ferguson reports that available ton miles are the best unit of output.[1] Ton miles sold are not a suitable output, since the short-term costs of an airline vary primarily with capacity rather than with units sold.[2] Considering that the airlines are mainly in business to haul passengers, not cargo, Stratford[3] feels passenger seat miles are a better measure of output, subject to a comfort (size/type of seat, noise level, seats per toilet, etc.), range, and speed index. Both Stratford and Schriever note that with today's exceptionally high rates of potential productivity associated with fast jet aircraft, the unit of time takes on greater significance.[4] Although the direct cost per seat mile may be comparable with earlier aircraft, the operating cost and potential profit per hour is considerably higher. Thus, today we have increasing incentives to reduce ground maintenance and service time. In the case of the new 20 million dollar 747 aircraft, we are seeing a changing era involving support equipment and maintenance policy.[5] To be profitable, the firm must have a high utilization rate.

Miller finds that the "Big Four" (American, Eastern, Trans-World, and United) appear to obtain economies of scale from the concentrated use of flying equipment and maintenance support facilities.[6] Cherington shows that the "Big Four" are inherently more economical in terms of operating cost per unit of output.[7] However, a large airline, compared to a middle size (Delta, Braniff, Western), may suffer some diseconomies of scale (increased cost of management overhead).

If the output of commercial aircraft is difficult to define, military aircraft output is more so. Ferguson in 1963 reported that due to lack of recorded data, an economist could not properly evaluate the output of military aviation.[8] Several years later (1971), Enthoven and Smith reported that the military can quantify the aviation output parameters through the units of potential sorties (adjusted for lethality of ordnance), loiter time, and air crew proficiency.[9] Donaldson and Blake consider the military output of fighters to be effective missions (measured in sorties) and ground alert time.[10] Hitch and McKean

report that military aircraft generate two primary outputs -- flying hours or sorties, and aircraft in commission -- in a production function situation "analogous to others encountered by economists in industry."[11] Sutton reports that, based upon recent 3M maintenance data, a Cobb-Douglas or CES (Constant Elasticity of Substitution) type of production function relationship exists between world-wide reports (ship and shore) of U.S. Naval aircraft inputs and the output of readiness (measured in aircraft ready hours).[12]

Gilster and Woodman, when investigating the relationship between the Air Force's use of labor and capital to generate output (flying hours), determined that an exponential (Cobb-Douglas) relationship appeared to exist between the inputs and the outputs.[13]

The inputs which produce the airline/military aircraft outputs can be classified by a number of categories. Ferguson felt the primary inputs were capital, flight-crew labor, and fuel.[14] Stratford reported three types of inputs: (1) standing cost, including interest, depreciation of assets, and insurance; (2) flying cost, including crew, fuel, maintenance and overhead cost; and (3) other costs, which represent a special cost to specific operations, such as time of departure and route selection, and cost of speed.[15] In the military sector, Sutton used the inputs of aircraft, maintenance man-hours, and spare parts.[16] In Lockheed ASW analysis of the input-output situation, the following categories of inputs were used: aircraft, maintenance personnel, support equipment, and spare parts.[17] Hitch and McKean in their analysis of the military aircraft production function considered that the major inputs were aircraft, maintenance equipment, maintenance personnel, and spare parts.[18]

All of this research in the airline and military sectors points toward a Cobb-Douglas production function type situation. With the exception of Sutton's work, none have quantified the elasticities of the actual inputs. No research has been done to establish what relationships may exist between the various management policies and output, or how this production function relationship may change under the constraints found aboard ship.

For the purposes of this study, the measurable outputs of sorties (U_1) and aircraft ready hours (U_2) will be used. The four inputs -- aircraft (W_1), maintenance personnel (W_2), support equipment (W_3), and spare parts (W_4) -- will be used with several specific maintenance and operating policies.

The Aircraft Input (W_1)

When designing an aircraft to meet certain goals, it becomes apparent that an exact or ideal solution is not possible. Each of the desired performance characteristics has a feedback or interrelation effect on the others. According to Corning, aircraft design is not an exact science but an iterative, cut-and-dry process.[19] Ultimately, the cost of commercial aircraft depends on take-off weight, wing area, thrust, payload, range, cruise speed, and weight of fuel.[20]

Military aviation is interested not only in cost but cost relative to the probability of combat mission success. The more common military aircraft characteristics which have a bearing on mission success are:

Speed (basic, combat and maximum at various altitudes)
Rate of climb at various altitudes
Ceiling
Payload range
Combat time, loiter time
Takeoff and landing characteristics

Military designers have established several cost estimating relationships (CER's) between the basic airframe variables.

Batchelder[21] reports the following labor hour CER:

$$H_{100} = 1.45W^{0.74}S^{0.43}$$

where

H_{100} = labor hours to produce the 100^{th} airframe
W = gross takeoff weight in pounds
S = maximum sea level speed in knots

Boeing[22] reports a CER which reflects both material and labor via a learning curve which is:

$$C = kW_eN^{0.7}V^2$$

where

C = total system cost
W_e = weight empty in pounds
V = maximum speed at sea level in knots
N = number of aircraft to be produced in the specific production run
k = a constant for the level of technology

More elaborate CER's have been established that consider each of the primary military variables, including such items as "electronic, complexity factors."[23]

In the Boeing case, doubling the output from 10 to 20 units reduces the average labor cost per unit from .49 to .37, a reduction of about 32 percent. Average fixed cost (research and development, tooling, etc.) declines also as a greater number of units are produced. All this points to the conclusion that aircraft manufacturing is a declining cost industry or a "natural oligopoly."

But after a "best" military aircraft has been designed, considering the various tradeoffs in relation to cost for various size production runs, we still do not have a military "revenue" or implicit value of having a specific aircraft to produce an output. Now the concept of a revenue function for tactical aircraft is difficult to define, compared to the airline industry.

No groups of measures of return (fighter/kill probability, attack bombs on target, ASW standard loiter time, etc.) are readily quantifiable or relate to a common metric. The ultimate logic for acquiring and maintaining a weapon system is to provide a combat capability. There are secondary reasons also, such as national prestige and the maintenance of a technology/manufacturing base.

The full expected return from a particular aircraft can not be satisfactorily described by any direct measure, such as bombs dropped, missiles fired, etc., since the ultimate value of the system depends on the particular scenario and national goals at the time of use. However, at some point in the decision process the planners and political reviewers (congressional and executive) had a choice among competing weapon systems and other social alternatives. Thus, the net value[24] of an "on going" system, such as current naval aircraft, should be at least equal to its future investment cost, otherwise some other system or social alternative should have been selected. For this reason the basic "revenue" return for a specific aircraft during its average life should be considered as being at least equal to or as great as the total investment cost of the aircraft.

Then, for a given maintenance policy (this type of policy changes the stream of rework and overhaul cost over a period of years), the revenue of a military aircraft can be considered as a function of the annual investment cost of a unit equivalent (U. E.)[25] aircraft which contains the following elements:

Flyaway unit
Initial spares
Initial ground support equipment
Support aircraft (overhead aircraft to handle training, pipeline and attrition)
Overhaul (engine) and rework (airframe)
Aircraft engineering changes
Expected life of the aircraft

A sample illustration of the investment cost determination of a U. E. aircraft month[26] will be helpful to illustrate the aircraft's implicit revenue value:

Row	Type of Cost	Aircraft A-4E
(1)	Flyaway cost	$ 600,000
(2)	Engineering changes	100,000
(3)	Pipeline factor, 20 percent of rows (1) and (2)	140,000
(4)	Operating life	150 months
(5)	Attrition factor, .4 percent/mo. or 42 percent of sum of (1), (2), (6)	367,500
(6)	Training factor, 25 percent of (1), (2)	175,000
(7)	Initial support equipment, 5 percent of (1), (2), (6)	43,750
(8)	Initial spares, 15 percent of (1), (2), (6)	131,250
(9)	Overhaul and rework, 20 percent of (1), (2), (6)	175,000
		Sum $1,732,000 for 150 months or $11,550/ month

This DoD method of costing a Navy U. E. aircraft includes support and spares for both the shore establishment and the ship. For our decision purposes only the ship U. E. cost will be considered. To do this we must "back-out" about 60 percent of the support equipment and at least one-third of the investment in spare parts. This gives an adjusted monthly cost for the A-4E of $11,080. From this it can be said that the "basic" revenue for the A-4E aircraft at sea is at least equal to $11,080 per month, or is an A-4E aboard ship is not flyable, it has an implicit opportunity cost of $11,080 per month. This measurement of cost will be the basic aircraft input for the A-4E of the model in section 6 and it will be indicated as ${}_2C_1$.

There are other bundles of cost which might measure more accurately the cost of the aircraft for some decision purposes. Should the U. E. aircraft be charged for attrition that may never occur? Shouldn't attrition be an operating cost like fuel and oil and expensed to the time period involved? Such a revised U. E. cost would be $8,800 per month and will be labled ${}_1C_1$. Since in this study the A-4E or other aircraft are being costed only as part of the aircraft carrier weapons system, shouldn't the operating life of the aircraft be limited to the period it is expected to be aboard a major carrier and not include that portion of life that the aircraft will be a part of the reserve fleet? If this approach is used, the cost should be amortized over only about 2/3 of the 150-month total life span of the airplane. In this case the A-4E monthly cost would increase to $16,625 and this will be labled ${}_3C_1$. Finally, shouldn't that portion of the investment cost which occurs as a stream of money over several years be discounted? This would result in a present value (PV) concept of aircraft cost. Attrition, engineering changes and overhaul/rework, when discounted at 10 percent, result in a PV U. E. monthly cost

for an A-4E of $6,333. This type of cost will be labled ${}_4C_1$. Thus for the first input (W_1) we have a range of different costs (${}_1C_1$, ${}_2C_1$, ${}_3C_1$, ${}_4C_1$) to represent several assumptions regarding the items that make up this total investment over time. Analysis will indicate whether the "best" solution of the objective function is sensitive to variation in the airplane unit cost.

Maintenance Manpower Input (W_2)

Personnel are required for mission preparation, routine servicing, gassing, loading of ordnance, etc. This type of labor cost is directly related to the number of sorties, except some types of sorties (i.e., mass flights of three times a day) cause peaks in manpower requirements that conflict with efficient labor utilization. Then manpower is necessary to perform scheduled maintenance. The frequency of scheduled maintenance and the complexity of this maintenance is a function of maintenance policy. Finally, maintenance manhours are used to handle the unscheduled maintenance or "fix it" items. The frequency of this type distribution is random for the various subsystems of the aircraft around some measurable mean. The mean occurrence of failures or malfunctions is a function of sortie rate, deck turn-ups, total flying hours, and type of mission (altitude, airspeed, etc.). Within limits, by increasing the amount of test and support equipment, management can reduce the average manhours required to service a discrepancy. To a degree, the maintenance manpower required can be decreased through the liberal use of throw-away components and spare parts. Thus there are trade-offs between the inputs of men, equipment, and spares to generate a specific sortie rate or aircraft ready level. But, the problem of comparing trade-offs is complicated because certain resources cannot be used concurrently with others and some resources are best employed without interruption (the policy of preemption) until the job or batch lot is finished. Limited access to certain sections of the aircraft can place an upward limit on the usefulness of additional personnel for certain discrepancies.

In most cases, not only the total number of men available affects performance but also the skill levels and the distribution of the men among the various shops. Since most of the airline and military maintenance data records indicate only aggregate manhours used, it is difficult to distinguish between workers on a basis of skill or to measure the extent to which substituting skilled for semiskilled personnel is economical. Interviews with military and civilian sector maintenance supervisors indicate that both suspect that productivity per manpower dollar would significantly increase if skill levels were upgraded through more training and if turnover were decreased through higher wages. On the other hand, the military organizations may be using enlisted maintenance manpower assigned as a "free good". According to one critic, the services tend to follow Parkinson's law and "the primary determinant of manhours to maintain aircraft may simply be the number of personnel assigned to that unit." [27]

In the military sector the average number of maintenance manhours required per mission (sortie) appears to be primarily an exponential function of the speed of the aircraft (index of complexity). [28]

Gilster and Woodman have reported that for the newer aircraft in the civil sector the ratio of labor/material is declining.[29] This implies a sensitivity to rising cost of labor (labor costs are rising faster than material costs) resulting in labor saving maintenance provisions being incorporated in the newer aircraft systems.

Now with this understanding of the maintenance personnel required to handle an average workload of an aircraft for various levels of complexity per mission, we still have described only part of the problem. Unfortunately, malfunctions and demands for maintenance manpower have a wide distribution about their mean and if personnel capabilities are just adequate to handle the average workload at times, long queues will develop of high cost aircraft or aircraft components. Therefore the relative "cost" of aircraft awaiting maintenance personnel must be compared to the relative "cost" of having greater manning insurance.

At this point an example of how the military estimate the cost of maintenance personnel is appropriate. Once the level of expected sorties or flight hours per month has been specified (say 20 sorties per month) the estimated average man months of support per aircraft is calculated. Consideration is given to complexity factors, time/motion studies, etc. The calculations are by types of skills (electricians, mechanics, hydraulic technicians, etc.) on a weighted average of pay grades (within a career skill no consideration is given for the degree of individual training).

With an estimate of the average type of man months required for each type of aircraft, an average monthly pay rate for these men is determined. This DoD maintenance wage rate considers utilization, retention, training and logistic support (commissary, medical care, messing, etc.) per man. The base pay that the sailor receives is only about 63 percent of this cost.[30] In the case of the A-4E maintenance man, the rate (${}_1C_2$) is \$419/month.

But these DoD manpower costs do not consider the investment cost of providing maintenance personnel accommodations aboard the space-limited aircraft carriers. Improved new ship standards require at least 28 square feet per sailor (berthing, messing, laundry, and sanitary facilities).[31] A recent naval study of the average cost of shifting and operating one square foot of space from storage to living space places the lower end of the cost estimate at about \$4 per ft^2/month.[32] Thus, the monthly cost of a maintenance man, for some decision purposes, is his monthly pay plus at least \$112 (28 ft^2 x \$4). In the case of the A4E maintenance man this revised rate (${}_2C_2$) is \$531/month.

Then if we are thinking of the maintenance personnel as coming from all-volunteer armed forces, or the full social cost of today's pay rates being less than market prices, the DoD monthly pay rates are low per type of skills. Attempts to estimate the "true" market value of military manpower have been made. The present enlisted wages appear to be only 60 percent of the amount which men of like age, education, skill, etc., could earn in civilian life.[33] It was further postulated that without the draft to induce enlistments we would have insufficient "true" volunteers to meet military needs at the present pay scale. If the amount of pay is a prime factor in obtaining suitable military maintenance manpower, current DoD monthly planning wage rates are low and do not reflect

the total national cost. Further, this distortion in pay can place the wrong incentives on investment decisions between military human capital and hardware capital. This may explain the Gilster and Woodman finding that military aircraft operations tend to be less capital intense relative to the airline operations.[34]

For any analytical cost trade-off between manpower, support equipment, and spares, the aircraft sensitivity comparisons should be made for a range of wage rates. The range used in this study will be from the low DoD figure ($_1C_1$) to a possible "full" cost rate that would reflect both shipboard accommodation cost and portion of the open market, full-wage value of trained technicians (at least an extra 40 percent over basic military pay rates).[35] The upper limit ($_3C_2$) for an A-4E maintenance man is $782/month. The results of this effort will assist the decision maker in knowing the direction he should move in spending his next increment of resources (on labor or on hardware) to obtain the largest increase in output and whether our current "low" military wage rates are leading to distortions in investment decisions.

Support Equipment (W_3)

The maintenance operation depends heavily on flight line support equipment and maintenance support equipment. In the private sector there is concern with the relative increasing cost of support equipment. Increased facilities and handling tools must be provided as newer aircraft become heavier and more complex. Support equipment costs are rising much faster than aircraft unit costs.[36] Also machine tools and facilities for component overhauls are larger and more complex. Today support equipment is a major airline cost problem.[37] Military maintenance equipment requirements per aircraft have been doubling about every ten years.[38] Avionics support needs have been growing faster than this. It has been estimated that the capital investment in support equipment now exceeds the investment cost in spare parts for new aircraft.[39]

This all implies the need for maintenance planning in order to obtain an effective utilization plan for facilities, tools, and equipment. Neither the civilian or military aircraft manager can tolerate unnecessary high cost aircraft delays due to limited or improper support equipment.

The airframe manufacturer identifies the basic support equipment requirements and their cost associated with his particular airplane. The user of the aircraft (airline or military) then analyzes his requirements in terms of the specific level of operations envisioned. Consideration is given to the number of aircraft involved (size of fleet), flight frequencies or estimated sortie rates, and the maintenance concepts. Scheduling and queueing problems are evaluated concerning the use of common support equipment for several types of aircraft. Unfortunately, standardized equipment is not keeping up with changes in technology, and the "jungle" of specialized support equipment grows.[40] We have for each type of aircraft, special pre-oilers, engine stands, hydraulic test stands, and hydraulic jacks, to name a few.

In the armed forces, we categorize support equipment into three types, with the spares for each: common equipment, such as that used to refuel, rearm, or tow aircraft; special support and maintenance equipment associated with specific aircraft; and training devices related to the aircraft weapon system.

In the past, the investment in support equipment amounted to about 7 percent of the value of the U.E. aircraft.[41] Today this may be equal to or greater than the sum spent for aircraft spare parts - up to about 20 percent of the cost of aircraft.[42]

Support equipment "saves" on the manhours required for scheduled and unscheduled maintenance, fueling and rearming of aircraft. Some support equipment permits the ship to repair replaceable parts, thereby decreasing the need for spare parts but at the cost of more maintenance manhours.

For this study, measurements will be taken from a sample of key support components (58 items, on the average, as reported monthly in the 3M reports), and this information will be used to estimate the inventory characteristics of the total population of the support equipment. The life of the original equipment is limited to an estimated 36 months by obsolescence, necessary modifications, and wear beyond tolerance. From the 3M data and from machine printouts of the total inventory of support equipment, per ship, per type aircraft, estimates of the unit cost and spares have been made. The cost of these inputs are labeled ${}_1C_3$ (without a space charge) and ${}_2C_3$ (with a charge for space). For the A-4E, the cost for ${}_1C_3$ is \$299/month and for ${}_2C_3$, \$379/month.

The Spare Parts Input (W_4)

The management job of spares is a major area of interest to the airlines and the services. In Navy Air, the investment value of spare parts stock is about \$2.15B. Navy annual additions to spare inventories (replacement for consumption, obsolescence, and modifications) is about \$122.2M. This is approximately 17.5 percent of the value of current military aircraft.

In the airline sector, spare provisioning for newer aircraft (747, 727, L500) is becoming a significant cost item. The airline investment cost in spares now runs between 14 to 18 percent of the basic airframe cost (depends partly on fleet size) and about 10 percent of the airline direct operating cost.[43]

The management of spares has many facets. This includes essentiality rules, replenishment rules, repair decisions, size of pools, and transportation and storage policies. Each of these sub-areas has its own implicit costs and payoffs. The issues involving levels of repair and speed of transportation will be discussed in more detail in the maintenance policy section.

Various theoretical systems have been developed which describe the requirements of an "optimal" inventory system. Such spare management decisions usually require inputs of the cost of reordering, stock depletion cost, and expenses associated with holding an item in inventory. Of course, the distribution of demand must be understood. With the management of military aircraft spares, we have also the cost of obsolescence and the cost of modifications to update spares. This can be a dominant spare parts' cost with today's changing technology.[44]

It is important to keep in mind that the inventory models are but limited abstractions of the real world. Spares management is concerned with future requirements and the expected distribution of demand for certain times. A major problem is the limited knowledge of the reliability characteristics of specific components.[45] Many have the traditional "U", a bathtub distribution of failures over time. The failure rate decreases at first — the "burn in" phenomenon — then the failure rate levels off at a fairly constant rate, and last we observe a marked increase in failures — the "wearout" region. Unfortunately, some parts do not appear to follow or exhibit any set "wear out" characteristics. With some, after "burn in", a constant failure rate continues indefinitely.[46] In this case any planned replacement policy is inappropriate. United Air Lines has found, after extensive test and many observations, that a constant failure rate (after burn in) is associated with many engine accessories, electronic units and hydraulic components.[47] To warrant replacement in advance of failure there must be an increasing failure rate over time and/or a penalty for a failure while in service.[48] Many components meet this criteria and should and do have a replacement policy. Other components should not be replaced until they fail.

For Naval aircraft carriers the problem of representing the distribution of demand has been handled pragmatically, by selecting a demand function on the basis of neatness of fit and analytic practicality. In general, the Poisson type distribution is used since the real life frequency of demand distribution is skewed to the left, like a Poisson, and its one parameter, distribution, requires only the mean rate of average use for estimation purposes.[49] A compound Poisson (with two parameters of distribution) can handle demands that have an exaggerated skewed distribution. The compound Poisson can also handle demands that occur in clusters or bursts.

The ordering policy is dictated by the management situation (requirement for essentiality, safety stock, deterioration of stock, etc.) and considers such items as usage rate, ordering cost per order, carrying cost, shortage cost, and reorder lead time. The common policy used is the (s, S) situation. In this case, when the stock, X, falls below a predetermined level, s, an order is placed for S-X units. S and s are chosen to minimize cost. A special case of (s, S) policy is the (S-1, S) situation. This establishes a one-to-one ordering and is used where high cost spares, such as aircraft engines, are involved.

Whatever model is used, the neatness of its fit to the real world is based upon a comparison to past data. Real time demands do not always follow past distributions, and this causes poor fits between the model and actual data. In addition, Poisson or exponential failure rate models do not meet the demand data for some types of aircraft components. As noted earlier, some equipments do not exhibit wearout characteristics and may have a random constant failure rate. Some parts are mainly liable to damage during installation or failure due to improper or "over" maintenance.

Haber has shown that the Naval aircraft spare parts inventory model does not fit actual usage too well. Over a two-year period, he has found that for several Naval aircraft, including the F-4 and A-4, 70 percent of the spare parts inventory are "slow" movers and that the demand for parts "depends on variables other than flying hours."[50]

However, for this study, the current aggregate inventory aboard the deployed aircraft carriers will be accepted as the basic input for spare parts (W_4). Statistical approximations will be used to determine full allowance of spare parts by type of aircraft. Estimates have been obtained, via visits to the ships and the Naval Support Activity, Washington, D.C., of the dollar value and cubic space taken up by a full inventory. Obsolescence continues to be a problem for static inventories. A 100-month time period appears to be a reasonable upper estimate of the life of parts before obsolescence or necessary modifications terminate the value of an unused inventory. In addition to the cost of having an inventory, there is an implicit saving due to the reduction of cannibalization actions that do not occur when the inventory is on hand. In the aggregate, each unit of inventory "saves" about $130/month in cannibalization cost.

The sum total of all this is that the cost that will be used for the W_4 (spares) input of the model in Section V can be expressed as a net monthly cost per unit of inventory. This input can be charged or not charged for the space that the inventory occupies. Sensitivity tests will later show that the cost of space is an important ingredient in determining the best inventory level. The cost of a unit of inventory without a charge for space will be labeled ${}_1C_4$; with a space charge, it will be ${}_2C_4$. For the A-4E the cost of ${}_1C_4$ is $381/month and of ${}_2C_4$, $476/month, respectively.

THE MAINTENANCE POLICY INPUT

The replacement or repair of deteriorating or failed equipment is a major consideration in the theory of maintenance. Gilster reports that the maintenance manhours required per flight hour increases noticeably as a result of aging after about 80 months of military aircraft life.[51] This increases the cost of operation and decreases the potential output. When a component fails or scheduled maintenance occurs, the objective is to restore the equipment to its almost original state as soon as possible. The objectives of an aircraft maintenance policy are to prevent deterioration of the aircraft fleet while at the same time meeting acceptable safety standards.[52] Of course, one must accomplish this protection and improvement of availability at acceptable levels of cost or within the resource constraints of the situation.

United Air Lines, ATA, and the Comptroller General of the United States all appear today to be asking the same question, "Are scheduled component overhauls really necessary?"[53]

Gilster and Woodman report that a large percentage of military component failures are induced due to "over-maintaining" with excessive number of inspections and overhauls.[54]

But old habits or established maintenance policies are not changed easily. To some, the GAO-suggested changes in maintenance policy are revolutionary ([change overhaul inspection schedule to one based on usage (flight hours) instead of calendar days, lengthen the overhaul interval, and use phased maintenance instead of complete overhauls]). The airlines use phased or "on-condition" maintenance (scheduling, as required, separate components for overhaul between flights or during other than heavy operating periods)

with intensive inflight monitoring to detect marginal units (e.g., compressors) in advance of failure. The commercial sector has found that they can both reduce maintenance cost on many systems and increase flight reliability through a policy of repairing only when necessary rather than performing preventive repairs.[55]

Although much of the literature concerning maintenance policies centers around the merits or lack of merit of periodic maintenance, other aspects of maintenance decisions may substantially affect the size of the maintenance labor force for a fixed level of output. These areas are:

(a) Cannibalization -- What is its value, and when should it be used? What is its true cost (manhours to remove and later replace, increased failure rate due to the additional "burn in", etc.)?

(b) Preemption (stopping work on one job in order to complete another) -- What is its worth, and when should it be done? From an engineering viewpoint, some repairs are best done sequentially without interruption until completed; other items are best repaired in batch lots.

(c) Discrepancy corrections -- when should flight discrepancies (which are not a safety-of-flight issue) be corrected (i.e., deferred to the end of the flying day)?

(d) COD -- carrier onboard delivery (COD) policies (all parts delivered to the ship within 5 days, 10 days, 15 days, etc.) can ease the needs for spare parts inventory, but this service has the increased cost of larger COD systems (both in aircraft-COD-inventory and operating costs of these support aircraft).

The total range of possible maintenance policies is large. For the purposes of this study, the marginal cost associated with three maintenance policies, other than the base case, will be quantified and ranked in order of importance and impact. The three variations used will be phased maintenance, changes in the priority of discrepancy corrections, and changes in the cannibalization rules.

THE SORTIE POLICY INPUT

The sortie policy for the military services or the scheduling policy of the airlines is a pacing item that sets the initial stage of events. For the airlines, the times of peak passenger demand are between 9 and 11 AM and 6 and 9 PM, when about 75 percent of the U.S. domestic flight departures occur.[56] This means, at best, a poor utilization of the total airline system investment (aircraft, airfields, FAA airways, etc.).

The military have a similar scheduling problem. A judicious assignment of mission times could level the peaks in support equipment and maintenance manpower requirements. However, combat demands (close air support, increased lethality of visual bombing systems, tactical targets of opportunity, etc.) are much greater during daylight hours, and a hostile air environment often favors using large-scale, surge type operations, during which you can be assured of having air superiority. Thus, the demands for both military and airline outputs conflict with the goals of ideal scheduling, dispatching, and use of support equipment. At most, our large military tactical air investment is fully

utilized only 12 hours per day (average daylight hours). In the private section 75 percent of the passenger departures occur during two daily spurts of activity which total only 8 hours.

To the military commander a great variety of sortie or "frag" (U.S. Air Force terminology) policies or patterns are possible, each with a wide range of ordnance/fuel configurations. For the Navy, the attack carrier usually operates aircraft for approximately 12 hours and is then off for 12 hours. Normally, the carrier flying is done in seven or eight "cycles" consisting of 1-1/2 to 1-3/4 hours each. For surge operations we might have only three major sortie groups per flying day or a longer flying day. For 3 typical CVA's deployed on a combat line at the same time, figure 1 illustrates three types of basic sortie policies and the effect each type of policy has on the sortie rates and availability.

In addition to the basic sortie policy (cyclic operation, daylight hours only, large mass flights only, etc.), there are specific sortie policy issues such as the following:

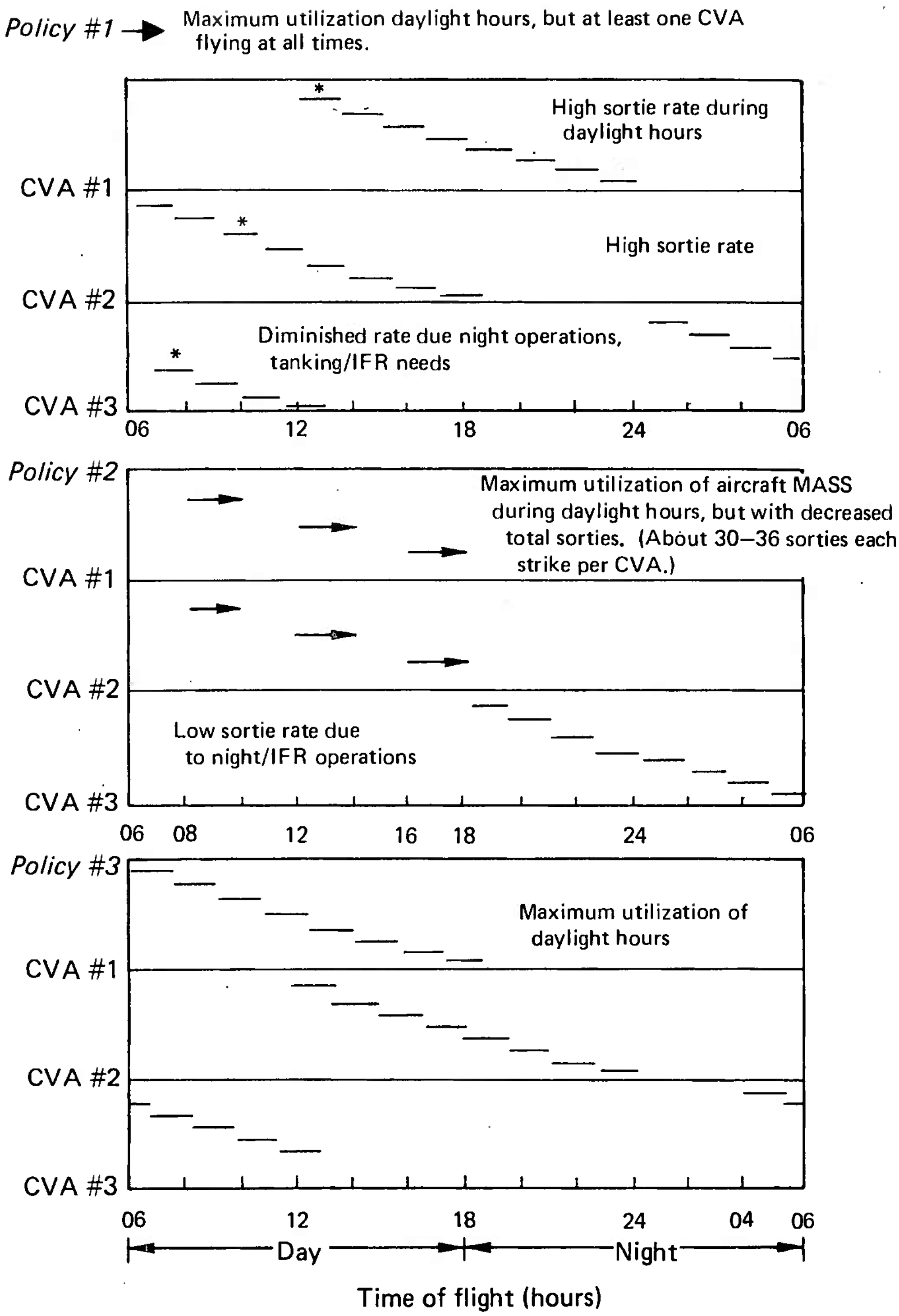
(a) Ground or deck alert (in the catapults, manned, etc.) versus air alert for AEW (Airborne Early Warning) and CAP (Combat Air Patrol) fighters.

(b) Spare aircraft policy. What is our required ratio of configured, manned back-up aircraft to scheduled aircraft? Can we count "known" up aircraft returning to the carrier from the previous cycle as the "spares" for the next cycle?

(c) Cancellation and substitution policy. When availability is low, what lower priority events can be cancelled to meet the desired combat output? Can we substitute some fighters for attack missions or substitute deck alert aircraft for airborne CAP?

Air Force studies have shown that these specific policy issues can have a great effect on the total level of combat output of an airwing.[57]

Because of the impact of policy in determining aircraft availability and the number of sorties that can be potentially generated, sortie policy (somewhat like maintenance policy) affects the needs of all resources - aircraft, support equipment, spares and maintenance personnel. Research fails to indicate that any metric or costing precedure has been previously established which quantifies the relative value of one sortie policy over another. However, the basic 3M data along with the results obtained by questionnaire and interview will give an indication of the cost and benefits of varying the flying day. From this, the commander can use a sortie policy "scale" to decide if the relative utility between each policy versus the cost or opportunities foregone appears to warrant such action.



**One and one-half hour sorties.*

FIG. 1: EFFECTS OF POLICY ON SORTIES

FOOTNOTES

[1] Allen R. Ferguson, "Commercial Air Transportation in the U.S.", in Studies in the Structure of the American Economy, edited by Wassily Leontief, et al, New York: Oxford University Press, 1963, pp. 421-423. Available seat miles are converted to ton miles and added to the ton miles of cargo actually carried.

[2] Ibid., p. 421.

[3] Alan H. Stratford, Air Transport Economics in the Super-Sonic Era, New York: St. Martin's Press, 1967, pp. 15, 36, 329.

[4] Bernard A. Schriever and William W. Seifert, Air Transportation 1975 and Beyond: A Systems Approach, Cambridge: MIT Press, 1968, p. 363.

[5] Laurence Doty, "Giant Jets to Force New Concepts, "Aviation Week and Space Technology, Nov 20, 1967, pp. 39-41, 44.

[6] Ronald E. Miller, Domestic Airline Efficiency, Cambridge: MIT Press, 1963, pp. 13, 14.

[7] Paul W. Cherington, Airline Price Policy: A Study of Domestic Airline Passenger Fares, Boston: Harvard University, 1958, pp. 42-50.

[8] Ferguson, op. cit., p. 421.

[9] Alain C. Enthoven and K. Wayne Smith, How Much Is Enough? New York: Harper & Row, 1971, pp. 155, 156, 218, 219.

[10] T. S. Donaldson and G. W. Blake, Aircraft Performance Related to Ground Alert Duration, RM-3628-PR, Santa Monica: RAND Corporation, April 1966, pp. 26-28.

[11] Charles J. Hitch and Roland N. McKean, The Economics of Defense in the Nuclear Age, Cambridge: Harvard University Press, 1963, pp. 266-268.

[12] S. Scott Sutton, A Study of Aviation Resources and Readiness Relationship, Washington, D.C.: Center for Naval Analyses, June 1970, pp. 1-2.

[13] Herman L. Gilster and Lloyd Woodman, "An Investigation into the Use of Labor and Capital for Aircraft Maintenance in the Military and Commercial Sectors," an unpublished memorandum from USAF Academy (Department of Economics) Colorado, May 1969, pp. 1-5.

[14] Ferguson, op. cit., p. 421.

[15] Stratford, op. cit., pp. 65-81.

[16] Sutton, op. cit., pp. 1-2.

[17] Lockheed Company, "P-3C Maintenance Simulation Model Description," an unpublished report prepared for the U.S. Naval Air Systems Command, Washington, D.C., 27 June 1969, pp. 21-24.

[18] Hitch and McKean, op. cit., pp. 267-268.

[19] Gerald Corning, Supersonic and Subsonic Airplane Design, Ann Arbor: Edward Brothers, 1963, p. 2:1.

[20] Ibid., p. 2:2.

[21] C. A. Batchelder, et al, An Introduction to Equipment Cost Estimating, RM-6103-SA, Santa Monica: RAND Corp., Dec 1969, p. 85.

[22] Maurice Olsen and John Adams, Analysis of Categories of Cost, Military Aircraft, Seattle: Boeing Airplane Co., 1961, pp. 1-5, 1-6.

[23] For amplification of military aircraft CER's, see Planning Research Corporation, Methods of Estimating Fixed Wing Air Frame Cost, Washington, D. C. April 1967; W. E. Mooz, The B-X: A Hypothetical Bomber Study, RM-4635-PR, Santa Monica: RAND Corporation, July 1965; and Donald M. Fisk, Estimates of Aircraft Characteristics with Some Implication for Cost Analyses, P3836, Santa Monica: RAND Corporation, April, 1968.

[24] The net value or utility of a weapon system would be primarily its military value; however, such a defense system has unique economic value to certain segments of the U. S. economy. The "congressional" type market place establishes the basis for a net national value through paying "premiums" for defense plant locations, splitting up production runs to help local investment problems, etc. These premiums have national utility, or our decisions to obtain these production options would lack a rational approach. Thus, the price paid for an airplane must at least equal its expected military plus domestic economic utility.

[25] The benchmark for DoD aircraft investment estimates is the U. E. aircraft. These are aircraft on the line assigned to a squadron, or for this study aboard an aircraft carrier. The U. E. aircraft is then "charged" for its share of other aircraft (overhead) that support the line unit, such as pipeline, training, and attrition airplanes. Source: J. A. Sullivan, "Aircraft Costing," Center for Naval Analyses (INS), Washington D. C. unpublished document of 20 Nov 1970.

[26] The period of one month is used since the data on the other inputs, which will be compared to investment cost (such as spares, maintenance, manpower, etc.) are recorded on a monthly basis.

[27] T. S. Donaldson and A. F. Sweetland, The Relationship of Flight Hours to Maintenance Manhours to Aircraft Flying Hours, RM-5701, Santa Monica: RAND Corporation, Aug 1968, p. 24.

[28] W. E. Mooz, op. cit., pp. 11-12.

[29] Gilster and Woodman, op. cit., p. 17.

[30] Secretary of the Navy, Retention Task Force Report, Washington, D. C.: U. S. Navy 1966, p. 347.

[31] L. F. Hicks, "The Aircraft Carrier and Its Air Wing: A Combat System," Naval Engineers Journal, June 1968, pp. 469-470.

[32] Naval Air Systems Command, Aeronautical Requirement: Level of Repair for Aeronautical Material, General Program Requirements, Washington, D. C.: U. S. Department of the Navy, March 1970, pp. 37-38 (mimeographed). On the same scale the cost of additional dry storage is $\$2/ft^2$/month and the cost of support/shop space is $\$40/ft^2$/month.

[33] U. S. Government, The President's Commission on An All-Volunteer Armed Force, Washington: U.S. Government Printing Office, Feb 1970, pp. 6-7.

[34]Gilster and Woodman, op. cit., p. 20.

[35]Military maintenance personnel who support new high technology aircraft can transfer directly and with ease to the airline industry. Present first-tour reenlistment of this type of personnel may be less than 6 percent. Interviews with systems analysis representatives of United, PAA and American Air Lines indicate they recruit trained personnel from the military to meet a great portion of their maintenance manpower needs, due to the pay differentials between the two aircraft industrial sectors.

[36]Harold D. Watkins, "747 To Intensify Airport Space Problems," pp. 39-41 and pp. 54-55, Aviation Week and Space Technology, Nov 20, 1967.

[37]M. Doyle, "Facilities, Equipment and Tools to Support Wide Bodied Airplane Maintenance in the 70's," an unpublished Boeing Co. paper delivered at the 6th Annual International Maintenance Symposium, Dec 8-10, 1970, Tulsa, Oklahoma, sponsored by FAA.

[38]L. F. Hicks, op. cit., pp. 465-466.

[39]Lockheed Co., "Preproduction Cost Project Techniques," ATA Maintenance and Management Conference, Nov 1969, Seattle, Washington, pp. 7-8 to 8-1.

[40]John T. Henrizi, "Diminishing Returns in the CVA," U. S. Naval Institute Proceedings, Aug 1964, pp. 76-77.

[41]M. E. Mooz, op. cit., p. 35.

[42]Lockheed Company, "Preproduction Cost Projection Techniques," op. cit., p. 30.

[43]W. D. Perreault, "Preproduction Cost Projection Techniques," Lockheed Georgia Co., an unpublished paper presented to the ATA Maintenance and Engineering Conference, Nov 1969, Seattle, Washington, pp. 7, 25, 30.

[44]James W. Peterson and Wilbur A. Stager, Design Change Impacts on Airframe Inventories, P-1055, Santa Monica: RAND Corporation, Oct 1957, pp. 11, 13, 14.

[45]Chaucey F. Bell and Milton Kamins, Planned Replacements, P-3052, Santa Monica: RAND Corporation, Jan 1965, p. 304.

[46]Ibid., p. 8.

[47]P. A. Hassey and S. G. Thomas, "Are Scheduled Component Overhauls Necessary?" an unpublished paper presented at the S. A. E. Aero and Space Engineering and Manufacturing Meeting, Los Angeles, Oct 5-9, 1964, on file at the FAA Library, Washington, D. C.

[48]Bell and Karmis, op. cit., p. 3 and Richard E. Barlow and Frank Proschan, Mathematical Theory of Reliability, New York: John Wiley, 1968, p. 47.

[49]Harrison S. Campbell, "Procurement and Management of Spares," in Defense Management, op. cit., pp. 189, 190.

[50]Sheldon E. Haber, "A Comparison of Usage Data Among Types of Aircraft," Naval Research Logistics Quarterly, Sept. 1967, pp. 404, 408.

[51]Herman L. Gilster, "A Statistical Analysis of Aircraft Maintenance Cost," Operations Research, Sep-Oct 1970, p. 912.

[52] F. S. Nowlan, Manager United Air Lines Operations Research, "Review of 'On Condition' Philosophies from a Planning and Operational Viewpoint," a paper presented to the 15th meeting of IATA Production and Control Subcommittee, Belgrade, Yugoslavia, Oct 7-9, 1969, pp. 2-3 of text.

[53] For expansion of this controversial point, see: F.S. Nowlan, Ibid.; P.A. Hussey, "Are Scheduled Component Overhauls Necessary?" a paper presented to the Society of Automotive Engineers, National Aero and Space Engineering and Manufacturing Meeting, Los Angeles Oct 5-9, 1964; Bell and Kamins, op. cit., p. 9; and, GAO, "Potential for Savings in Aircraft Maintenance (Military), B-15260," May 1970, p. 6.

[54] Gilster and Woodman, op. cit., pp. 23-24.

[55] T. S. Donaldson and A. F. Sweetland, The Relationship of Flight-Line Maintenance Man-Hours to Aircraft Flying Hours, RM-5701-PR, Santa Monica: RAND Corporation, Aug 1968, pp. 22-23.

[56] Aviation Week and Space Technology, Nov 20, 1967, p. 53.

[57] Allen S. Ginsberg and Barbara A. King, Base Operations - Maintenance Simulator, RM-4072-PR, Santa Monica: RAND Corporation Sep 1964, pp. 2, 3, 17.

SECTION III

THE CONCEPT OF THE AIRCRAFT CARRIER PRODUCTION PROCESS AND ITS ABSTRACT MODEL TO HANDLE THIS PROCESS

THE DEVELOPMENT OF THE BASIC MODEL

The aircraft carrier production process is a transformation of materials into products (sorties, availability, etc.) by a series of energy applications, all at a cost.

The inputs of this process are indicated by the symbols W_j ($j = 1, 2, \ldots, n$) and are measurable quantities of goods and services consumed in the production process: aircraft, labor, support equipment, and spares. These inputs also have a decision or scenario dimension, such as maintenance and sortie policies; this type of dimension is indicated by the symbols X_k and Y_ℓ respectively.

The outputs of the process are denoted by U_i ($i = 1, 2, \ldots, m$) and are the measureable economic goods or services produced in the process. They are such items as aircraft ready hours, sorties, and aircraft in commission, all of which occur in a joint production process. The outputs have a degree of coupling between each other that is a function of the product qualities of U_i and/or the joint quantities of the inputs. This point will be expanded upon later in this section.

In the general case we can consider U_i as a function of n variable $W = (W_1, \ldots, W_n)$, and we can call this a production function.

Using symbols, the production function is:

$$
\begin{aligned}
U_1 &= f_1\,({}_1W_1, \ldots, {}_1W_n;\, X_k,\, Y_\ell\,) \\
U_2 &= f_2\,({}_2W_1, \ldots, {}_2W_n;\, X_k,\, Y_\ell\,) \\
&\vdots \\
U_i &= f_i\,({}_iW_1, \ldots, {}_iW_n;\, X_k,\, Y_\ell\,) \\
&\vdots \\
U_m &= f_m\,({}_mW_1, \ldots, {}_mW_n;\, X_k,\, Y_\ell\,),
\end{aligned}
\tag{3.1}
$$

where

${}_1W_1$ = the amount of the first input used for output U_1

$\vdots$

${}_mW_n$ = the amount of the nth input used in the output U_m .

In equation (3. 1), f_i represents the form of a relationship or dependence of outputs $U = (U_1, \ldots, U_m)$ on inputs W. Available amounts of inputs (different from actual amounts consumed in the process), degree of technology, and time are also factors that affect f_i. Only the inputs to the left of the semicolon (;) in equation (3. 1) are fully variable in quantity.

The possible degree of coupling that may exist between U_1, U_2, U_3, etc. can be expressed as a functional relationship such as:

$$F (U_1, U_2, U_m) = 0 \qquad \text{(coupling within an output sector)} \qquad (3.2)$$

or

$$F ({}_1W_n, {}_2W_n, \ldots, {}_mW_n) = 0 \qquad \text{(coupling within an input vector),} \qquad (3.3)$$

which is independent of W in (3. 2) or independent of U in (3. 3) above. [1]

In either case we desire to maximize the outputs - $U_1, U_2, \ldots, U_n$ - subject to the constraints

$$\sum_{j=1}^{n} W_j \leq b_i \text{ (inputs cannot exceed assets), and}$$

$$\sum_{i=1}^{m} U_i \geq R \text{ (certain types of requirements must be met).}$$

It is assumed that the factors (W_j) are always organized in a technically optimal fashion so that the production functions are defined as giving the maximum amount of output possible for any factor combination. Thus, where the partial derivatives, $U_i' = \frac{\partial U}{\partial W}$, become negative, we have inefficient points, and these regions are excluded from the production function surface. The set of efficient points is where $U_i > 0$ and $\Delta U_i / \Delta W_n \geq 0$.

The inputs of this aircraft carrier (CVA) production process (W_j) consist of four general categories:

W_1 = aircraft in units
W_2 = maintenance labor in months of manpower (160 hrs/man/month)
W_3 = maintenance support equipment in units of allowance (a standard is established for each type of aircraft)
W_4 = spare parts in units of allowance (a standard is established for each type of aircraft).

The outputs (U_i) then are derived from an energy transformation process (see figure 2), where we have four separate but interacting sequences (IP_1, IP_2, ..., IP_4). Assuming that each process has a range within which we have factor substitution, the process production functions have the form:

$$U_i = \gamma(S_{24}, S_{34}, W_{14}, W_{24}, W_{34})$$

$$S_{12} = \lambda(S_{21}, S_{31}, -S_{13}, W_{21}, W_{31}, W_{41})$$

$$S_{13} = \zeta(S_{21}, S_{31}, -S_{12}, W_{21}, W_{31}, W_{41})$$

$$S_{24} = \epsilon(S_{32}, S_{12}, -S_{23}, -S_{21}, W_{12}, W_{22}, W_{32}, W_{42})$$

$$S_{23} = \pi(S_{32}, S_{12}, -S_{21}, -S_{24}, W_{12}, W_{22}, W_{32}, W_{42})$$

$$S_{21} = \tau(S_{32}, S_{12}, -S_{23}, -S_{24}, W_{12}, W_{22}, W_{32}, W_{42})$$

$$S_{34} = \upsilon(S_{23}, S_{13}, -S_{31}, -S_{32}, W_{13}, W_{23}, W_{33}, W_{43})$$

$$S_{32} = \chi(S_{23}, S_{13}, -S_{31}, -S_{34}, W_{13}, W_{23}, W_{33}, W_{43})$$

$$S_{31} = \psi(S_{23}, S_{13}, -S_{32}, -S_{34}, W_{13}, W_{23}, W_{33}, W_{43})$$

where S_{12} = processing factor from sequence 1 to 2

S_{34} = processing factor from sequence 3 to 4

and W_{12} = input factor 1 (aircraft) to sequence 2 (see figure 2)

⋮

W_{43} = input factor 4 (spare parts) to sequence 3.

The integrated production function for the process becomes:[2]

$$U_i = \gamma(\lambda, \zeta, \epsilon, \pi, \tau, \upsilon, \chi, \psi, W_1, W_2, W_3, W_4) = \Theta(W_1, \ldots, W_4) \quad . \quad (3.4)$$

The "best" expansion of the process is determined by minimizing total cost for various parameters of U_i. As we expand this aircraft carrier production process, the corresponding cost function could pass through five phases. For sufficiently small U_i there will be idle capacity in all processes (IP_1, ..., IP_4). At a certain level of output, one of the four capacity factors will be fully utilized; at a later point two capacities will be exhausted, etc. Each time a capacity limit is reached, the partial derivative will equal zero for the factor in question, and the marginal cost function will be continuous but kinked at those capacity points.[3]

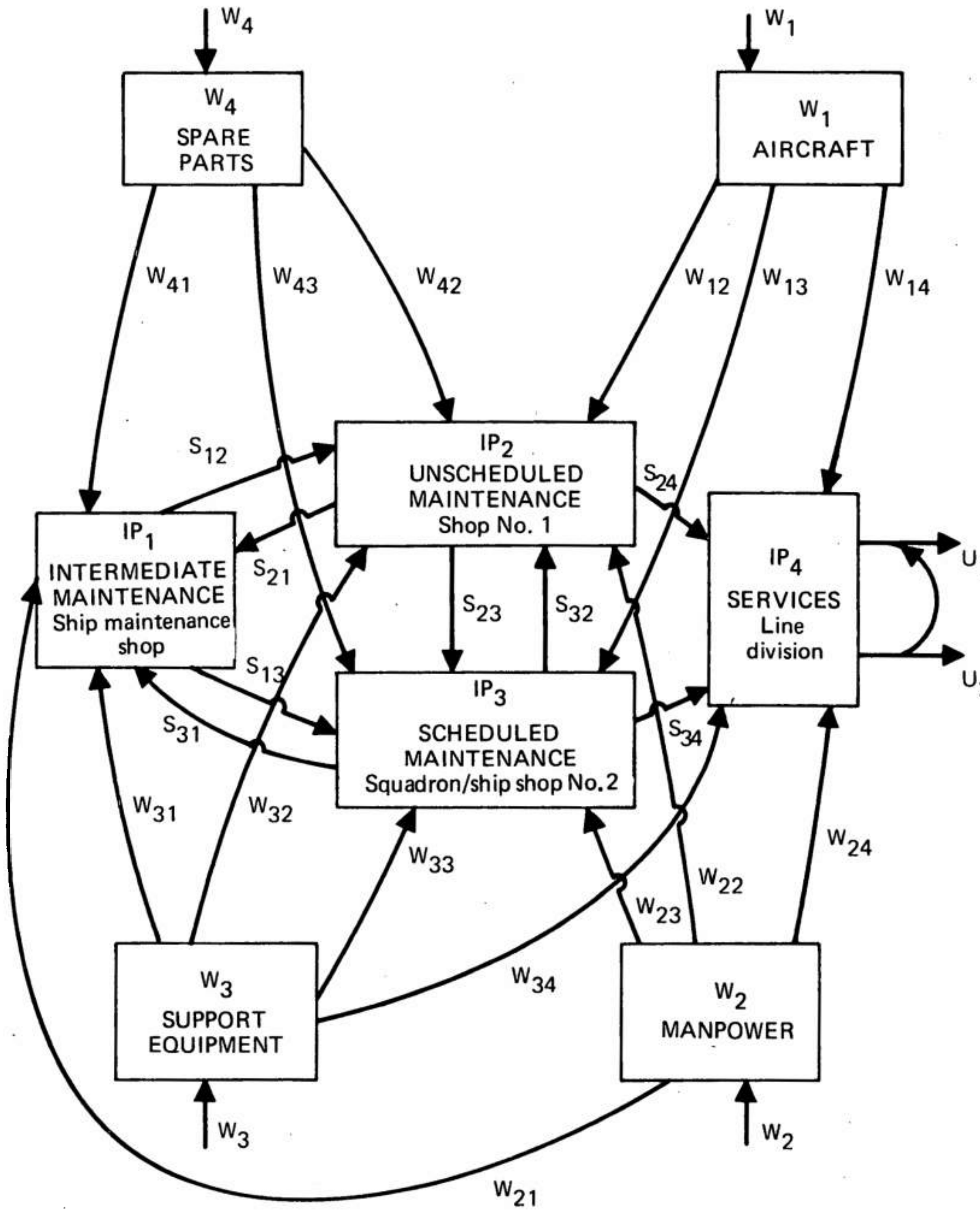


FIG. 2: THE MAINTENANCE TRANSFORMATION PROCESS

After recording the initial input output data (involving the aircraft carrier) and comparing the relationship of outputs (U_i's) to each other, the possible outputs of "aircraft in commission," "availability,"[4] and "aircraft ready hours" actually turned out to be but one output expressed with different scales. Thus the aircraft carrier production process has but two outputs - sorties (U_1), and availability or aircraft ready hours (U_2).

But there is a negative correlation between availability and the sortie rate. Increasing the sortie rate tends to drive availability down.[5] As the number of sorties goes up, the time the aircraft is down increases for support actions and unscheduled maintenance actions. High availability means a higher number of flights could be flown (you have to have an aircraft up before it can be flown); availability will go down or be consumed when the increased sorties are flown. In an investigation of U. S. Marine aircraft (F-4, A-4), Guthrie and Means indicated that the relationship between sorties and ready hours is of a negative exponential type.[6] Analysis of the 3M data for the five aircraft observed in this research shows the "best" fit to be an arc of a circle (see figure 3) rather than a negative exponential.

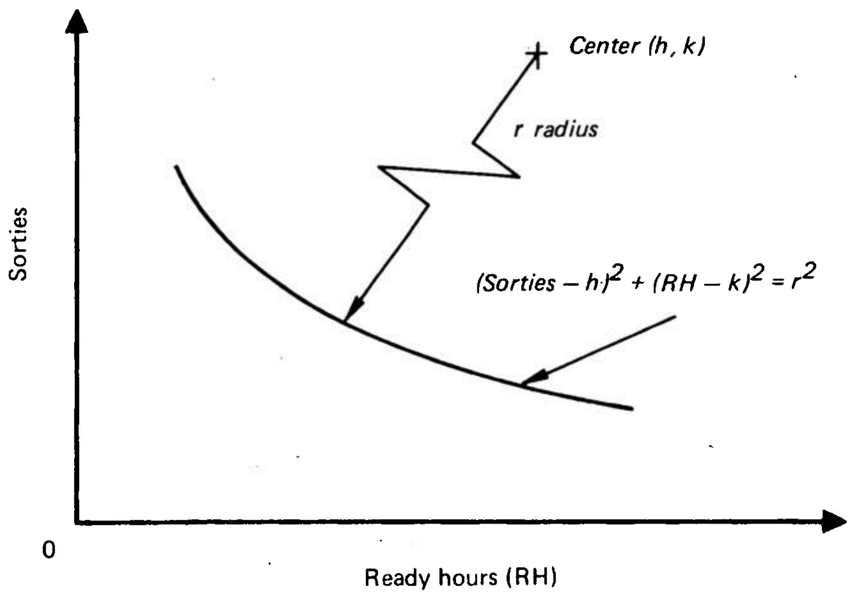


FIG. 3: RELATION BETWEEN READY HOURS AND SORTIES

This means that if all inputs (W_1, W_2, W_3, W_4) are held constant and sorties are increased, ready hours (U_2) above the required residual level are consumed, or some Δ function of (U_2) is an input of U_1 (a degree of coupling).

Recall that equation (3.1) was reduced to two outputs (U_1, U_2), which gives:

Sorties $$U_1 = f_1(W_1, W_2, W_3, W_4; X_k, Y_\ell)$$

RHrs $$U_2 = f_2(W_1, W_2, W_3, W_4; X_k, Y_\ell)$$

Then holding ready hours constant ($\bar{U}_2$), the net sortie effect is a production function of the type:

$$\bar{U}_1 = f_1(W_1, W_2, W_3, W_4; X_k, Y_\ell) + |f_2|\,\Delta(W_1, W_2, W_3, W_4; X_k, Y_\ell), \tag{3.5}$$

where

$\bar{U}_1$ = total potential sorties from both U_1 and U_2 inputs

f_1 = relationship or function between sorties and sortie inputs

$$|f_2| = \frac{dU_1}{dU_2}$$

$$\Delta(W_1 \dots W_4; X_k, Y_\ell) = \left[f_2(W_1 \dots W_4; X_k, Y_\ell) - \bar{U}_2\right],$$

which represents the potential functional increase or decrease in sorties obtainable from consumed ready hours (U_2) above or below the base case ($\bar{U}_2$).

The calculations of $|f_2|$ and $\bar{U}_1$ are accomplished as follows:

Obtain the first derivative of the function relating to U_1 and U_2, namely

$$(U_1 - h)^2 + (U_2 - k)^2 - r^2 = 0 \ ,$$

$$\frac{d\,3x^2}{dx} = 6x\frac{dx}{dx}$$

which can be written

$$U_1^2 - 2hU_1 + h^2 + U_2^2 - 2kU_2 + k^2 - r^2 = 0$$

$$\frac{\partial U_1}{\partial U_2} = \frac{1}{2}\cdot\frac{1}{2}\left(-4U_2^2+8KU_2-4K^2+4r^2\right)^{-1/2}\cdot\left(-8U_2+8K\right) \quad \tfrac{1}{4}$$

$$U_1 = +\frac{2h}{2} \pm \frac{1}{2}\sqrt{-4U_2^2+8kU_2-4k^2+4r^2} \quad .$$

Now let

$$Z = -4U_2^2 + 8kU_2 - 4k^2 + 4r^2 \quad .$$

Then

$$\frac{dU_1}{dZ} = \pm\overset{1/4}{\frac{1}{2}}Z^{(-1/2)}, \text{ where } U_1 = a \pm \frac{1}{2}z^{(1/2)}$$

and

$$\frac{dZ}{dU_2} = -8U_2 + 8k \quad ;$$

but

$$\frac{dU_1}{dZ} = \pm\frac{1}{2}Z^{(-1/2)} = \pm\frac{1}{2}(-4U_2^2+8kU_2-4k^2+4r^2)^{-1/2}.$$

Since

$$\frac{dU_1}{dU_2} = \frac{dU_1}{dZ} \times \frac{dZ}{dU_2} \quad , \quad \tfrac{1}{4}$$

$\therefore$ Substituting $$\frac{dU_1}{dU_2} = \left(\pm\frac{1}{2}\right)(-4U_2^2+8kU_2-4k^2+4r^2)^{(-1/2)}(-8U_2+8k)$$

$$= \pm 2(-U_2+k)(-U_2^2+2kU_2-k^2+r^2)^{(-1/2)} \quad .$$

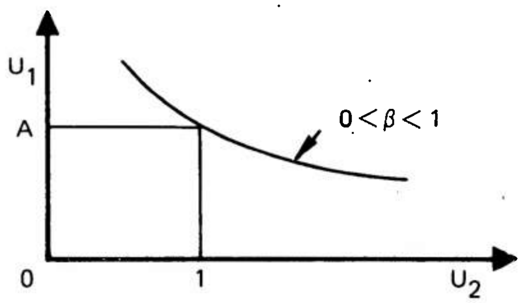
However, the region of economic feasibility in our allocation process does not permit the use of the negative roots from this derivative, therefore,

$$\bar{U}_1 = f_1(W_1 \ldots W_4; X_k, Y_\ell) + 2(-U_2+k)(-U_2^2+2kU_2-k^2+r^2)^{(-1/2)} \cdot \left[f_2(W_1 \ldots W_4; X_k, Y_\ell) - \bar{U}_2\right] \quad (3.5a)$$

Although the following alternate function has not been used, it is developed for completeness. Within the range of observed values of U_1, U_2, the values of the arc $\left[(U_1-h)^2 + (U_2-k)^2 - r^2 = 0 \right]$ can be approximated by a double-log transformation, such as described by Johnston,[7] of the type

$$\log U_1 = \alpha - \beta \log U_2 \quad ,$$

where $\log A = \alpha$ and A is the value of U_1 when $U_2 = 1$, as shown below;



But the expression $\log U_1 = \alpha - \beta \log U_2$ may be written as

$$U_1 = AU_2^{-\beta}$$

and

$$\frac{dU_1}{dU_2} = - A\beta U_2^{(-\beta-1)} \quad ,$$

thus, giving this relationship;

$$\bar{U}_1 = f_1 (W_1 \ldots W_4; X_k, Y_\ell) + A\beta \left[f_2 (W'_1 \ldots W'_4; X_k, Y_\ell) \right]^{(-\beta-1)}$$

$$\times \left[f_2 (W'_1 \ldots ; . \; Y_\ell) - \bar{U}_2 \right] \quad , \qquad (3.5b)$$

where

$\bar{U}_1$ = total potential sorties from both U_1 and U_2 inputs

f_1 = relationship or function between sorties and sortie inputs

f_2 = relationship or function between ready hours and ready hour inputs

W_j = sortie inputs

W'_j = ready hour inputs

A = the value of U_1 where U_2 = 1 for the function of $U_1 = AU_2^{-\beta}$

β = the exponential constant obtained by regression analysis to fit the relationship between U_1 and U_2 for n = 1, 2, ... T observations. For our observations the domain of β is $0 < \beta < 1$.

In addition, it seems reasonable intuitively to expect these different variations of the production function (3.5, 3.5a, 3.5b) to have the following properties:

(a) In the efficient region, an increase in the level of any input (W_j) should produce an increase in the level of output (U_i).

(b) Subsequent increases in the level of any one input, holding all other inputs constant, should produce smaller and smaller absolute increases in the level of output.

(c) The marginal increase in output resulting from an increase in any input will be greater if other inputs are also increased.

(d) Many combinations of inputs can be used to produce the same level of output - the concept of an isoquant.

THE OBJECTIVE FUNCTION WITH ITS CONSTRAINTS AND ASSOCIATED ASSUMPTIONS

Having established the general properties of the desired production function, it appears that the Cobb-Douglas (C-D) function (as expanded by Tintner) meets the requirements. Specifically, the Tintner (C-D) is[8]

$$U = \alpha_0 W_1^{\alpha_1} W_2^{\alpha_2} W_3^{\alpha_3} W_4^{\alpha_4} \epsilon \quad , \qquad (3.6)$$

where

U = output in units or dollar value

α_0 = a scaling efficiency or technological change factor

W_j = inputs in units of use or dollar value

α_j = elasticity with respect to the W_j input

ϵ = a multiplicative error term: It is assumed that $\epsilon \geq 0$.

In applying function (3.5) to the aircraft carrier series of energy/resource applications, the objective function now becomes one of maximizing U_1;

$$U_1 = \alpha_0 W_1^{\alpha 1} W_2^{\alpha 2} W_3^{\alpha 3} W_4^{\alpha 4} + \left| f_2 \right| \Delta \; (\alpha_0' W_1'^{\alpha' 1} W_2'^{\alpha' 2} W_3'^{\alpha' 3} W_4'^{\alpha' 4}) \; ; X_k, \; Y_\ell , \tag{3.7}$$

subject to

(1) $W_j > 0$ (feasible region),

(2) $d_j \leq W_j \leq b_j$ (resource constraints)

(3) $\sum_{j=1}^{n} C_j W_j \leq C_t$, $t = 1, 2, 3, \ldots, p$ (cost restraint per unit of time),

(4) $\frac{\partial U_i}{\partial W_j} \geq 0$ (marginal productivity cannot be negative),

where

W_j are the resources in units of each type (j).

d_j & b_j represents the lower limit and the upper limit respectively of the physical constraints on the total weight or size (ft^2) of resources, by type (j).

α_0 is the coefficient to express the specific levels of technology.

α_j are the exponential coefficients associated with each input or the percentage change in output (U) for a given percentage change in W_j.

C_j is the unit cost of each resource in dollars, by type (j).

C_t is the upper dollar budget limit of all resources for a unit of time.

X_k & Y_ℓ represent two types of policy, a combined tactical decision or scenario dimension (X_k) (e.g., flying shall be continuous, 24 hours a day, in 1-3/4 hour cycles) with a combined maintenance decision (Y_ℓ) (fix all discrepancies as occurring, fly until failure occurs, etc.)

$\left| f_2 \right|$ dU_1/dU_2

Δ represents the potential increase or decrease in sorties obtainable above the policy required residual.

However, the issue of the type of possible coupling that might exist between the outputs and the specific inputs is not resolved here. Going back to the observed world, we are able to record independently, without any dominant functional relationship, the various levels of U_1 and U_2 for specific total levels of inputs (W_j). However, there is a relationship or coupling between each input vector (${}_1W_1$, ${}_2W_1$; ${}_1W_2$, ${}_2W_2$; . . . , ${}_1W_4$, ${}_2W_4$) and the outputs. For example, we observe the total labor that goes jointly to produce both U_1 and U_2 but not the amount of labor just to produce U_2. In summary, we observe the "rim" values of the following two simultaneous equations:

$$U_1^{\#} = \alpha_0 \, {}_1W_1^{\alpha_1} \, {}_1W_2^{\alpha_2} \, {}_1W_3^{\alpha_3} \, {}_1W_4^{\alpha_4} \, \epsilon_1$$

$$U_2^{\#} = \alpha'_0 \left(\frac{{}_2W_1}{W_1^{\#}}\right)^{\alpha'_1} \left(\frac{{}_2W_2}{W_2^{\#}}\right)^{\alpha'_2} \left(\frac{{}_2W_3}{W_3^{\#}}\right)^{\alpha'_3} \left(\frac{{}_2W_4}{W_4^{\#}}\right)^{\alpha'_4} \epsilon_2$$

where # = indicates a "rim" value that is actually observed

${}_1W_1$ = the cell input of W_1 to assist in output 1

.
.
.

${}_2W_4$ = the cell input of W_4 to assist in output 2

α_1 = elasticity with respect to the ${}_1W_1$ input

.
.
.

α'_4 = the elasticity with respect to the ${}_2W_4$ input

α_0, = the technology change factor with respect to output 1

α'_0 = the technology change factor with respect to output 2 .

The problem then is to simultaneously solve the above equations for the 18 unknown "cell" values (α_0, α_1, . . . , α'_4, and ${}_1W_1$, ${}_1W_2$, . . . ${}_2W_4$).

In natural logarithms, the joint production process is:

$$\ell\eta U_1 = \ell\eta\,\alpha_0 + \alpha_1\,\ell\eta\,{}_1W_1 + \alpha_2\,\ell\eta\,{}_1W_2 + \alpha_3\,\ell\eta\,{}_1W_3 + \alpha_4\,\ell\eta\,{}_1W_4 + \ell\eta\,\epsilon_1$$

$$\ell\eta U_2 = \ell\eta\,\alpha_0' + \alpha_1'\,\ell\eta\,{}_2W_1 + \alpha_2'\,\ell\eta\,{}_2W_2 + \alpha_3'\,\ell\eta\,{}_2W_3 + \alpha_4'\,\ell\eta\,{}_2W_4 + \ell\eta\,\epsilon_2 \quad .$$

A procedure to quantify the values for unknown cells or the specific amounts of inputs used to produce each of the two outputs and estimate the α_j's (elasticities of the inputs) is as follows:

(a) First, estimate the initial values for ${}_1W_1$, ${}_1W_2$, etc. such that they satisfy $W_1 = {}_1W_1 + {}_2W_1$, with all $W_j > 0$

$W_2 = {}_1W_2 + {}_2W_2$.

(b) Second, with the above initial values, obtain approximate estimates for the α_j's by a likelihood estimate methodology.

(c) Third, update the W_j's by a quadratic methodology.

(d) Fourth, update the α_j's by the likelihood estimate methodology.

(e) Continue this iterative process until a local minimum has been reached.

(f) Last, test for convergence or near convergence to a "best" local solution by varying the initial values used in step (a) over a wide range, and determine and compare the minimum solution arrived at by each set of starting values.

However, before dealing with this general case of four inputs, we will investigate a smaller example to show how the relationships behave.

In the small case there are but two inputs, capital (K) and labor (L), each with its coefficient of elasticity (α and β respectively).

In this case we have:

$$\ell\eta U_1 = \ell\eta\alpha_0 + \alpha_1\,\ell\eta K_1 + \beta\,\ell\eta L_1 + \ell\eta\,\epsilon_1$$

$$\ell\eta U_2 = \ell\eta\alpha_0' + \alpha_1'\,\ell\eta K_2 + \beta'\,\ell\eta L_2 + \ell\eta\epsilon_2$$

and we observe U_1, U_2; $K = K_1 + K_2$; $L = L_1 + L_2$ for each time period.

In addition, a few mild assumptions, must be made all of which appear consistent with operational experience. There is also strong empirical evidence that these assumptions and the resulting model accurately reflect real world experience.

(a) The distribution of U_i (i = 1, 2) is approximately normal with an expected value of U_i that is a linear function of K_i and L_i and has a variance that is independent

of K_i and L_i. This is the simplest case and, as will be seen later, appears to be supported by empirical evidence from the real world.

(b) The values taken on by K_k and L_i are predetermined.

(c) The successive values of U_i are independent of the prior values.

(d) $\ell\eta\epsilon_{ij}$ $(i = 1, 2)$, $j = 1, 2, \ldots n$, are normally distributed, with $E(\epsilon_{ij}) = 0$ and $Cov(\epsilon_{ij}, \epsilon_{ik}) = 0$ for all $j \neq k = 1, 2, \ldots n$, and $V(\epsilon_{ij}) = \sigma^2$ for all j.

THE DERIVATION OF A RELATED LIKELIHOOD FUNCTION

To describe the maximum likelihood (L) estimates of α_0, α_1, and β, we have:

$$\text{Max } L = \frac{1}{(2\Pi)^{T/2}\sigma_1 T} e^{-\left(\frac{1}{2\sigma_1^2}\left(\sum_{t=1}^{T}\left[\ell\eta U_1^t - \ell\eta\alpha_0 - \alpha_1 \ell\eta K_1^t - \beta \ell\eta L_1^t\right]^2\right.\right.} ,$$

where T = the number of observations obtained over time.

The maximum likelihood estimates of the variances is of the type:

$$\frac{1}{\sigma_1^T} e - \frac{1}{2\sigma_1^2} S ,$$

where

$$S = \sum_{t=1}^{T} [\ell\eta U_1^t - \ell\eta\alpha_0 - \alpha_1 \ell\eta K_1^t - \beta \ell\eta L_1^t]^2 .$$

Taking the natural logs we have:

$$-T \ell\eta\sigma_1 - \frac{1}{2\sigma_1^2} S .$$

Then taking the partial derivations of $\ell\eta L$ with respect to σ_1 we have:

$$\frac{\partial \eta L}{\partial_1} = \frac{T}{\sigma_1} - \frac{1}{2} S \cdot \frac{(-2)}{\sigma_1^3} = 0$$

which becomes

$$\sigma_1^2 = \frac{S}{T} .$$

So the minimized natural logs of the likelihood function for α_0, α_1, β is proportional to:

$$\ell n \sum_{t=1}^{T} \left[\ell n U_1^t - \ell n \alpha_0 - \alpha_1 \ell n K_1^t - \beta \ell n L_1^t \right]^2 \tag{3.8}$$

Now the determination of the maximum values of α_0, α_1, β etc. for the given K_i's and L_i's is straightforward. Of course, the maximum likelihood estimators are not necessarily unbiased for small samples. Analysis of residuals from actual data will bound somewhat the bias problem. Empirical evidence indicates that when the sample is 20 or more the bias is probably quite small. Similarly, we can obtain the maximum likelihood estimates of α_0', α_1', and β'.

THE DERIVATION OF A RELATED QUADRATIC SOLUTION

Unfortunately, as stated earlier, we observe only the total values of K and L, or $K = \Sigma K_i$ and $L = \Sigma L_i$. We are not able to record the individual K_i's or L_i's.

In this case we can solve for the K_i's by a sequential technique, moving toward a minimum value of equation (3.8). At that point the "best" approximate values will be established for α_0, α_1, β, etc., and the objective function (3.7) can be maximized, subject to the constraints of the aircraft carrier production function situation.

To solve for the K_i's, start with initial estimates for K_1^t, K_2^t, L_1^t, L_2^t that will satisfy:

$$K^t = K_1^t + K_2^t, \text{ where } K_i\text{'s and } L_i\text{'s} > 0$$

$$L^t = L_1^t + L_2^t \quad .$$

Then with these initial estimated values and the likelihood value established in (3.8), we can compute estimates for α_0, α_1, β, α_0', β' by regression analysis.

With the estimates of α_i's and β_i's and using our basic case (two inputs K, L, only), the revised values of K_1, K_2, L_1, L_2 will now exist, where

$$\ell n \sum_{t=1}^{T} \left(\ell n U_1^t - \ell n \alpha_0 - \alpha_1 \ell n K_1^t - \beta \ell n L_1^t \right)^2 + \ell n \sum_{t=1}^{T} \left(\ell n U_2^t - \ell n \alpha_0' - \alpha_1' \ell n K_2^t - \beta' \ell n L_2^t \right)^2$$

is a maximum with respect to α, β.

But $K^t = K_1^t + K_2^t$.

Now let $k_1^t = \frac{K_1^t}{K^t}$ (in order to have $0 < k_1 < 1$) and $\bar{k}_1^t = 1 - k_1^t$; then we have:

$$\text{Min} = \ell\eta \sum_{t=1}^{T} \left(\ell\eta U_1^t - \ell\eta\alpha_0 - \alpha_1 \ell\eta K^t - \alpha_1 \ell\eta k_1^t - \beta \ell\eta L_1^t \right)^2$$

$$+ \ell\eta \sum_{t=1}^{T} \left(\ell\eta U_2^t - \ell\eta\alpha_0' - \alpha_1' \ell\eta K^t - \alpha_1' \ell\eta \bar{k}_1^t - \beta' \ell\eta L_2^t \right)^2 .$$

But let

$$\tilde{U}_1^t = \ell\eta U_1^t - \ell\eta\alpha_0 - \alpha_1 \ell\eta K^t - \beta \ell\eta L_1^t$$

$$\tilde{U}_2^t = \ell\eta U_2^t - \ell\eta\alpha_0' - \alpha_1' \ell\eta K^t - \beta' \ell\eta L_2^t \;;$$

then

$$\text{Min} = \ell\eta \sum_{t=1}^{T} \left(\tilde{U}_1^t - \alpha_1 \ell\eta k_1^t \right)^2 + \ell\eta \sum_{t=1}^{T} \left(\tilde{U}_2^t - \alpha_1' \ell\eta \bar{k}_1^t \right)^2 \tag{3.9}$$

Taking the partial derivatives of (3.9) with respect to $\mathbf{k_i}$ and setting this equal to zero, we have

$$\frac{\partial}{\partial k_i} = \frac{2(\tilde{U}_1^t - \alpha_1 \ell\eta k_1^t}{\sum_{t=1}^{T} (\tilde{U}_1^t - \alpha_1 \ell\eta k_1^t)^2} \left(\frac{-\alpha_1}{k_1^t} \right) + \frac{2(\tilde{U}_2^t - \alpha_1' \ell\eta \bar{k}_1^t)}{\sum_{t=1}^{T} (\tilde{U}_2^t - \alpha_1' \ell\eta \bar{k}_1^t)^2} \left(\frac{\alpha_1'}{\bar{k}_1^t} \right) = 0. \tag{3.10}$$

But $0 < k_1 < 1$.

Thus a series expansion of $\ell\eta k_1$ would be:

$$(k_1 - 1) - \tfrac{1}{2}(k_1 - 1)^2 + \tfrac{1}{3}(k_1 - 1)^3 - \tfrac{1}{4}(k_1 - 1)^4 + \tfrac{1}{5}(k_1 - 1)^5 \ \ldots$$

or the partial derivative value (3.10) becomes:

$$\frac{\left[\tilde{U}_1^t - \alpha_1 (k_1^t - 1)\right]}{\sum_{t=1}^{T} \left[\tilde{U}_1^t - \alpha_1 (k_1^t - 1)\right]^2} 2 \left(\frac{-\alpha}{k_1^t} \right) + \frac{\left[\tilde{U}_2^t + \alpha' k_1^t\right]}{\sum_{t=1}^{T} \left[\tilde{U}_2^t - \alpha_0' (\bar{k}^t - 1)\right]^2} \left(\frac{\alpha'}{1 - k_1^t} \right) = 0 .$$

Let

$$a = \sum_{t=1}^{T} \left[\tilde{U}_1^t - \alpha_1 (k_1^t - 1)\right]^2 \qquad (3.11)$$

$$b = \sum_{t=1}^{T} \left[\tilde{U}_2^t - \alpha_1' (\bar{k}_1^t - 1)\right]^2 \qquad (3.12)$$

multiplying we have:

$$\frac{-\alpha_1 \tilde{U}_1^t + \alpha_1^2 (k_1^t - 1)}{a k_1^t} + \frac{\alpha_1' \tilde{U}_2^t + \alpha_1'^2 k_1^t}{b(1 - k_1^t)} = 0$$

transposing and collecting terms we have:

$$a(\alpha_1'^2 k_1^t)^2 - b\alpha_1^2 (k_1^t - 1)^2 - b(1 - k_1^t)(\alpha_1 \tilde{U}_1^t) + a k_1^t \alpha_1' \tilde{U}_2^t = 0$$

which becomes a quadratic of the form.

$$k_1^{t^2} (a\alpha_1'^2 - b\alpha_1^2) + k_1^t \left[2b\alpha_1^2 + b\alpha_1 \tilde{U}_1^t + a\alpha_1' \tilde{U}_2^t\right] + (-b\alpha_1^2 - b\alpha_1 \tilde{U}_1^t) \qquad (3.13)$$

The values for a and b change little between iterations, as we converge to a solution, if T is sufficiently large (>20). Therefore the values of a and b from the prior iteration can be used for the present iterative, and we have, for each step, a near constant value for a and b for any specific iteration.

This quadratic (3. 13) has only one unknown k_1, for which an estimated solution can be obtained. We will accept only the root for k_1 that is positive and where $0 < k_1 < 1$. In a similar fashion, L_1 (labor) can be estimated.

With these revised values for K_1, K_2, L_1, L_2, updated regression estimates can now be made for α_0, α_1, β. This in turn updates the values of K_1, K_2, L_1, L_2. Thus, the sequential process continues until we have the best values for α and β, such that with the determined distribution between K_1, K_2, and L_1, L_2, the sum of the squares is a minimum. To test the possibility that this "final" solution may be but a local rather than a near absolute minimum, different initial estimates for K_1, K_2, L_1, L_2 can be made. Then, by the sequential process, if we reach the approximate same final values for α, β, K, L, etc., an absolute minimum solution has most probably been established. If several local minima are established (the expected result in this case), a fairly exhaustive search will be made to determine, with a high probability of success, which of several subsolutions are unwanted stationary points and which are the near "true" minimum values.

In this basic case a methodology was established to solve for K, L and α, β; but in the general case we have:

$$\ell\eta U_1 = \ell\eta\alpha_0 + \alpha_1 \ell\eta_1 W_1 \ldots \alpha_4 \ell\eta_1 W_4$$

$$\ell\eta U_2 = \ell\eta\alpha_0' + \alpha_1' \ell\eta_2 W_1 \ldots \alpha_4' \ell\eta_2 W_4$$

which now becomes one of revising values for W_i's such that

$$\ell\eta\sum_{t=1}^{T} \left[\ell\eta U_1^t - \left(\ell\eta\alpha_0 + \alpha_1 \ell\eta_1 W_1^t + \alpha_2 \ell\eta_1 W_2^t \ldots \alpha_4 \ell\eta_1 W_4^t \right) \right]^2$$

$$+ \ell\eta\sum_{t=1}^{T} \left[\ell\eta U_2^t - \left(\ell\eta\alpha_0' + \alpha_1' \ell\eta_2 W_1^t + \alpha_2' \ell\eta_2 W_2^t \ldots \alpha_4' \ell\eta_2 W_4^t \right) \right]^2 \qquad (3.14)$$

is a minimum with respect to the α_i's.

In this case let

$${}_1Z_1^{\,t} = \ell\eta U_1^t - \ell\eta\alpha_0 - \alpha_2 \ell\eta_1 W_2^t - \alpha_3 \ell\eta_1 W_3^t - \alpha_4 \ell\eta_1 W_4^t$$

$${}_1Z_2^{\,t} = \ell\eta U_2^t - \ell\eta\alpha_0' - \alpha_2' \ell\eta_2 W_2^t - \alpha_3' \ell\eta_2 W_3^t - \alpha_4 \ell\eta_2 W_4^t \quad .$$

Then via the method that was developed in equation (3. 9),the problem becomes one of:

$$\text{Min} = \ell\eta\sum_{t=1}^{T} \left({}_1\tilde{Z}_1 - \alpha_1 \ell\eta_1 w_1^t\right)^2 + \ell\eta\sum_{t=1}^{T} \left({}_1\tilde{Z}_2 - \alpha_1' \ell\eta_2 \overline{W}_1^t\right)^2 \; .$$

By substitution ($W_1 = {}_1W_1 + {}_2W_1$; $w_1' = \frac{{}_1W_1}{W_1}$; $\overline{W}_1 = 1 - W_1$), obtaining the partial derivatives with respect to w_1', and finally in the collection of the terms the quadratic becomes:

$$w_1'^2 \left(\alpha_1'^2 - \alpha_1^2\right)^* + w_1' \left(2\alpha_1^2 + \alpha_{11}\tilde{Z}_1 + \alpha_{11}'\tilde{Z}_2\right)^* + \left(-\alpha_1^2 - \alpha_{11}\tilde{Z}_1\right)^* = 0 \quad .$$

*See footnote, p. 40.

This will establish the revised values for the ${}_1W_1$ and ${}_2W_1$. Making another pass through equation (3. 14) with ${}_2Z_1^{\,t}$ and ${}_2Z_2^{\,t}$, which stand for

$$ {}_2Z_1^{\,t} = \ell\eta U_1^{\,t} - \ell\eta\alpha_0 - \alpha_1\ell\eta_1 W_1^t - \alpha_3\ell\eta_1 W_3^t - \alpha_4\ell\eta_1 W_4^t $$

$$ {}_2Z_2^{\,t} = \ell\eta U_2^{\,t} - \ell\eta\alpha_0' - \alpha_1'\ell\eta_2 W_1^t - \alpha_3'\ell\eta_2 W_3^t - \alpha_4'\ell\eta_2 W_4^t \quad . $$

We can in turn establish the quadratic for W_2, which is:

$$ (w_2')^2(\alpha_2'^2 - \alpha_2^{\,2})^* + w_2'(2\alpha_2^{\,2} + \alpha_{22}Z_1 + \alpha'_{22}Z_2)^* + (-\alpha_2^{\,2} - \alpha_{12}Z_1)^* = 0 \quad . $$

Thus, by a series of sequential steps, the absolute minimum solution will most probably be approached which in turn will establish the "best" values for the α_i's. Returning to the objective function (3. 7), the output can be maximized, subject to the constraints for specific levels of policy (X_k, Y_ℓ).

*Indicates that these terms will also include a "near" constant value of the type

$$ a_i = \sum_{t=1}^{T} \left[Z_1 - \alpha_i(w_i^t - 1) \right]^2 $$

as described in equations (3. 11) and (3. 12).

SUMMARY

The aircraft maintenance transformation process for generating outputs (sorties and ready hours) has been developed and traced to where it became:

$$U_i = \theta(W_1, \ldots, W_4) \tag{3.4}$$

Then, if the ready hours output (U_2) is held constant, the net effect is a production function of the type:

$$U_1 = f_1(W_1, \ldots, W_4; X_k, Y_\ell) + |f_2| \, \Delta(W_1, \ldots, W_4; X_k, Y_\ell) \quad , \tag{3.5}$$

and the objective function becomes one of maximizing U_1 which is:

$$U_1 = [\alpha_0 W_1^{\alpha_1} \ldots W_4^{\alpha_4} + |f_2| \, \Delta(\alpha_0' W_1^{1^{\alpha_1'}} \ldots W_4^{1^{\alpha_4'}}]; X_k, Y_\ell \tag{3.7}$$

Subject to:

$$W_j > 0$$

$$dj \leq W_j \leq b_j$$

$$\sum_{j=1}^{N} C_j W_j \leq C_t$$

$$\frac{\partial U_1}{\partial W_j} \geq 0$$

where

W_j	are the resource inputs of each type (j)
d_j & b_j	represent lower and upper limits of the inputs respectively
α_0	is the coefficient to express the specific levels of technology
α_j	are the exponential coefficients associated with each input
C_j	is the unit cost of each resource
C_t	is the upper budget limit of all resources for a unit of time
X_k & Y_ℓ	represent two types of policies - maintenance and flying policies

$|f_2|$ dU_1/dU_2

Δ represents the potential increase or decrease in sorties obtainable above the required residual.

Unfortunately, in real life one cannot directly observe or record the amount of the individual inputs used to specifically produce U_1 (sorties) or U_2 (ready hours). The total amount of W_j that produces both U_1 and U_2 simultaneously is observed. We observe the "rim" values of the following two simultaneous equations:

$$
\begin{aligned}
U_1^{\#} &= \alpha_0 \, {}_1W_1^{\alpha_1} \cdots {}_1W_4^{\alpha_4} \\
U_2^{\#} &= \alpha_0' \, {}_2W_1^{\alpha_1} \cdots {}_2W_4^{\alpha'_4} \\
& \quad\;\; \overline{W_1^{\#}} \cdots \; \overline{W_4^{\#}}
\end{aligned}
$$

where # indicates the "rim" values.

The problem then is to simultaneously solve the above equations for the 18 unknown "cell" values $(\alpha_0 \ldots \alpha_4'$, and ${}_1W_1, \ldots {}_2W_4)$.

A procedure to quantify the values for the "cells" has been developed which in essence amounts to the following:

(a) First, estimate the initial values for ${}_1W_1, {}_1W_2$, etc., such that they satisfy —

$W_1 = {}_1W_1 + {}_2W_1$, with all $W_j > 0$

$W_2 = {}_1W_2 + {}_2W_2$.

(b) Second, with the above initial values then obtain approximate estimates for the α_j's by the likelihood estimate methodology.

(c) Third, update the W_j's by the quadratic methodology.

(d) Fourth, update the α_j's by the likelihood estimate methodology.

(e) Continue this iterative process until a local minimum has been reached or approached.

(f) Last, test for convergence or near convergence to a "best" local solution by varying the initial values over a wide range of starting values.

FOOTNOTES

[1]Ragnar Frisch, Theory of Production, (Chicago: Rand McNally, 1965, pp. 275, 276, 277; and Sven Dano, Industrial Production Models, (New York: Springer-Verlag, 1966), p. 183.

[2]For expansion of this integration concept, see Sven Dano, Industrial Production Models, New York: Springer Verlag, 1966, pp. 162-163.

[3]Sven Dano, op. cit., pp. 155, 163.

[4]Percent monthly availability is defined as the (number of aircraft days assigned) times (24) minus (not operationally ready hours due to supply, and scheduled and unscheduled maintenance) divided by (total aircraft hours assigned).

[5]Guthrie, Donald and Means, Edward H., "Relationships Among Potential Sorties, Ground Support, and Aircraft Availability," Naval Research Logistics Quarterly, Vol. 15, No. 4, Dec 1968, p. 497.

[6]Ibid., p. 498.

[7]J. Johnston, Econometric Methods, New York: McGraw-Hill, 1963, p. 48.

[8]Gerhard Tintner, "A Note on the Derivation of Production Functions from Farm Records," Econometrica, Vol. 12 (1944), pp. 28-29. In addition, a constant elasticity of substitution (CES) type function was evaluated. However, interviews with Sutton (of Sutton and Lloyd, op. cit.) and Brown (of Brown and Schwartz, op. cit.), during the interval of Feb-March 1971, indicated that the neatness of fit between a Cobb-Douglas and CES, using observed 3M data, has been about the same. Considering the computation advantages of the C-D function (considerable savings in computer time) over the CES, the decision was made to use a Cobb-Douglas as long as the coefficient of determination (R^2) is high.

SECTION IV

DATA FOR PRODUCTION FUNCTION ESTIMATION AND UTILITY VALUES FOR RESOURCE ALLOCATION

This section covers the three main categories of input data that are used to validate the postulated Cobb-Doublas type aircraft maintenance model: (1) 3M observations, (2) questionnaire information, and (3) the results from interviews and field trips.

THE MAINTENANCE AND MATERIAL MANAGEMENT (3M) DATA

The Navy has several times changed the 3M reporting format in recent years. The 3M data is considered to be unreliable for several months after each change because of delays and errors of interpreting the new instructions. As a result of field trips and some pretesting of the data, a total of 24 monthly observations (from July 1968 to October 1970 less the months of January to April 1970) appeared to be the best. Monthly 3M at-sea reports were then obtained on all large aircraft carriers (11 CVA's) and their embarked airwings for this same period. The 3M Headquarters personnel at Mechanicsburg, Pennsylvania, suggested that where possible the first and last months of each ship's cruise/maintenance data should be deleted. Pretesting indicates that these months (initial and last) and the month of December (long holidays and inport times) are probably different from the rest of the observations. Field trips to four aircraft carriers also indicated that the monthly status of spare parts (w_4), as reported, is less precise than the other inputs. In addition, the spare parts inventory status is not always updated each month; thus, where possible, we avoid using adjacent month observations aboard the same ship.

Appendix A indicates the total population of observations obtained by the actual aircraft carrier and airplane squadron, excluding the first and last cruise months plus December:

Type aircraft	Number of 3M monthly observations
F-4	143
A-7	93
A-6	59
A-4	50
E-2	58

From the above a sample of 30 observations from the A-6, A-4, and E-2 aircraft was drawn. Sixty observations (30 Atlantic, 30 Pacific) were used for the F-4 and A-7 aircraft. The raw data (some of it originally obtained in binary or alpha numeric format) was then processed so that for each observation there would be six pieces of information: the two outputs (U_1 - sorties, U_2 - ready hours), and the four inputs (w_1 - aircraft, w_2 - manpower, w_3 - support, w_4 - spare parts).

In addition to these observations, the basic 3M data permitted statistical tests for a number of possible relationships, such as spare parts versus cannibalization, number of "no defects" actions[1] versus support equipment, and number of flights versus level of ready hours.

The following relationships were noted:

Cannibalization - A high degree of correlation (.727) existed between the lack of spare parts and the cannibalization rate. On the average, an additional 6.9 cannibalization actions took place per squadron per month with a reduction of one unit of spare parts. Each cannibalization required two maintenance actions - take it out of one aircraft and then put it in another - with an average total labor cost per cannibalization of 4.85 hours. Thus for each decrease of a percent of spare parts of inventory, we expanded our labor needs 33.5 hours, equal to an additional monthly labor cost of about $130. At the very least this is an additional cost for not having a sufficient inventory of spare parts to meet demand.

No defects - Intuitive judgment and interview information indicated that there should be a correlation between the number of "no defects" and the amount of test/support equipment available to the squadron. Unfortunately, sufficient 3M data was not reliable in this area. Different squadrons with like aircraft report "no defects" rates ten or more times higher than a similar squadron with the same level of support equipment but on a different ship. Probably some units are interpreting the reporting requirements for "no defects" actions differently. In any event, no statistical conclusion can be drawn at this time regarding a possible relationship between "no defects" actions and the level of test equipment.

Spare parts for support equipment - The spare parts for the support equipment are in many cases different from the aircraft spares. The 3M data and our cruise reports do not report the inventory level of these spares, but the 3M data does indicate how often the support equipment is down for repairs, scheduled maintenance, and awaiting parts. On the average, the observed support equipment was down only 14 percent of the time, but when it was down, it was in this category 81 percent of the time because of a lack of spare parts.

Labor categories - Navy planning figures estimate a maintenance man month as having 120 productive hours out of a potential total of 160. For the observations used in this sample, the overtime ran an average of 24.6 to 28.0 percent with a few squadrons running a six-month average of 73 percent. Thus, 160 productive hours per man, which includes overtime, will be used instead of the normal 120. Of the total labor hours inputs (W_2) observed in the data, 35 percent of the hours were used for unscheduled maintenance and 18 percent were used for scheduled maintenance. These percentages will be used later when varying the W_2 input to consider the potential labor savings of certain maintenance policy changes.[2]

Ready hours versus sorties - Review of the literature, interview response, and field trip observations all indicated a negative correlation between sortie rates and aircraft readiness. As the sortie rate goes up availability goes down. As the number of sorties increases the time the aircraft is down for support action (fuel, oil service) directly increases and the probability of maintenance also increases. When an aircraft is in maintenance it is not available. Hence, increasing the number of daily sorties decreases the ready hours, if all other inputs are held constant. High availability means a higher number of flights could be flown; availability will go down when the increased sorties are actually flown. Actual observations of sorties versus ready hours for the F-4 aircraft are shown in appendix B. In this case the data tested fits an arc of a circle of the type

$$\underset{(\sigma_h)}{(X-h)^2} + \underset{(\sigma_k)}{(Y-k)^2} = \underset{(\sigma_r)}{r^2},$$

where X = ready hours (RH)

Y = sorties

h = X coordinate of the center of the circle

k = Y coordinate of the center of the circle

r = radius of the circle

(σ_i) = the standard error of this dimension of the arc.

In this case the F-4 arc is part of a circle with the following characteristics:

$$(RH-\underset{(1.8)}{140.1})^2 + (\text{sorties}-\underset{(3.1)}{206.3})^2 = (\underset{(3.4)}{208.8})^2$$

At the most frequently observed area (RH = 400 per aircraft or 56 percent availability), the "slope" or trade off between sorties and ready hours potentially required is .546. If the sorties are increased until RH = 300, the slope increases to approximately .600. If the sorties are decreased until RH = 500 the slope decreases to approximately .505. A linear approximation of this relationship does not fit as well as the arc. For the F-4 it is:

$$\text{Sorties} = 44.927 - \underset{(.127)}{.546}\ RH \qquad R^2 = 0.444$$

The R^2 for the arc $[(X-h)^2 + (Y-k)^2 = r^2]$ is 0.580

The slope for the other aircraft at their most frequently observed level of readiness is:

A-7 .646 at RH = 460

A-6 .395 at RH = 500

A-4 .797 at RH = 500

E-2 .658 at RH = 430

While it is difficult to be sure of the actual amount of shift in the slope from RH = 300 to RH = 500, all of the data and interview results suggest an arc is a better approximation of the relationship between sorties and ready hours than a linear function. We can be quite confident of the direction of shift in the slope but less confident in the amount of change.

QUESTIONNAIRE

A questionnaire, appendix C, was designed to obtain information not available in the 3M maintenance reports and to act as a check on the possibility of observer bias that might exist in the interviews and field trips. The questionnaire was pretested on six former aircraft carrier department heads stationed in the Washington area. To increase validity of the questionnaire, questions were inserted which could be verified through interview and secondary sources (cruise records). Reliability was checked through the degree of consistency between questionnaire and interview responses. The most factual data — 3M records and cruise reports — had a positive correlation with the questionnaire and interview information.

The questionnaire was sent to all of the larger Naval aircraft carriers not in shipyards for overhauls or repairs (total population 11). Eight replies were received. Field trips and interviews covered two of the nonrespondents. The information pattern from these nonrespondents appeared to be about the same as those who returned questionnaires.

Information obtained from these questionnaires will be used in section VI to assist in the interpretation of the results obtained from the Cobb-Douglas type production model. The key patterns for questionnaire response are as follows:

a. Question 5 - If 10 percent more aircraft were placed on the ship, would the total output increase or decrease? Two indicated some increase, six indicated a definite decrease in output.

b. Question 6 - If 5 percent fewer aircraft were on the ship, would the total output increase or decrease? One indicated slight decrease, four reported no change, one an increase of 5 percent and two an increase of 10 percent.

c. Questions 8, 9, and 10 - If supply of spare parts support equipment, and quality of maintenance personnel were increased by 10 to 20 percent, how would this affect the potential sortie output? The responses were:

		Amount of change in output			
Variable increased	Decreased	No change	+5%	+10%	+15%
Spare parts	0	3	-	2	1
Support equipment	0	2	2	2	-
Quality of people	0	2	2	1	1

d. The final question, number 11, was an open-ended one to indicate whether reasonable reliability (consistency) had been achieved and to determine whether any major issues had been overlooked. The replies were most informative. Three indicated the greatest need for change involved the internal ship communications (supervisors unable to communicate quickly to supply and maintenance spaces from the aircraft flight and hangar decks). Two indicated an urgent need for better handling of materials (conveyor

belts, dumbwaiters, etc.) and better shop spaces for support equipment. Two strongly indicated a need for more efficient, trained personnel. Finally, two would spend the next million dollars on spare parts and rotatable pool components to decrease the time lost awaiting maintenance action and awaiting parts and to reduce cannibalization.

INTERVIEWS, FIELD TRIPS, AND THE DETERMINING OF UTILITY VALUES FOR RESOURCE ALLOCATION

This section will cover interviews and field trips directly related to establishing the bounds or qualifying the 3M and questionnaire data.

Coast Guard. The Coast Guard Aircraft Maintenance Section stresses a high readiness condition (the static situation) more than the sortie rate. Given this emphasis it appears that the Coast Guard considers the Navy's planning levels for spare parts and support equipment low. Although no specific figures were suggested, it was inferred that Navy levels should be 10 to 20 percent higher. Where support equipment of a Naval type was in use by the Coast Guard, satisfactory availability rates could not be maintained using the Navy inventory model for spare parts, leading to the conclusion that the Navy was long on aircraft and equipment but short on spare parts.

Naval Air Systems Command. One conclusion drawn here was that an airline-type progressive maintenance policy (incremental maintenance following a sample plan for some components and on others only when indication of need or failure) instead of the Navy's present maintenance policy could save the Navy airwings 25 to 50 percent of their present expended labor on scheduled maintenance activities. In addition, a policy of deferring discrepancies not involving flight safety until the end of the flying day, where possible, would permit better use of maintenance personnel and shop facilities, thereby reducing nonscheduled maintenance labor by perhaps as much as 10 percent.

Carrier Division Five. Staff members stressed the need for improved or more reliable "yellow gear" (support equipment on the hangar and flight deck) and weapons handling equipment. During the period June to October 1970, CarDiv 5 staff monitored a program where an additional 41 million dollars of aircraft spare parts were shipped to deployed Pacific aircraft carriers. As a result, availability of all types of airwing aircraft improved. Readiness of the A-7 aircraft improved the most (up 19 percent), and readiness of the A-6 improved the least (up 4 percent). This suggested that the A-6 was already fairly well supported, and the marginal return for additional spare parts was small. It was noted that only about 20 percent of the present AVCAL inventory (spares) is ever used on a specific cruise. This may indicate the need for a better carrier aircraft spare parts inventory model and the need to concentrate more on the rapid turnover of spare parts.

Information gathered from visits to four aircraft carriers in the Mediterranean was consistent with the questionnaire and the 3M recorded data, except in the "no defects" area. It was postulated that a correlation would be established between a rise in "no defects" actions and the lack of fully equipped support equipment. Several ways were suggested to reduce the loss of time awaiting spare parts. The brute force method is to have a larger inventory of parts, but accurate inventory control with rapid electronic search of own and nearby ships for needed material and a quick air resupply from the United States may be more effective than just increasing the inventory.

The key personnel on all ships visited expressed the need for additional skilled labor. Better trained people are needed to handle maintenance and support equipment. Three of the four felt they did not need and did not want more total maintenance personnel, just the need to upgrade the quality of existing personnel.

The opinion was expressed that the ship's official deck multiple or aircraft load was too high to maximize sortie outputs, leading to unnecessary maintenance delays and queues. If the aircraft deck load were reduced by 5 percent, the total output for the remaining 95 percent of the aircraft was estimated to be equal to or slightly greater than that of the original situation.

In addition, it was confirmed that during the 3M period of interest (July 68 to Oct 70), the aircraft carriers in the sample had, in general, a full-deck multiple of aircraft aboard during their deployed months. This means that when the number of aircraft decreased (say 12 to 10.5) for any specific squadron, no squadron benefit resulted in possible reduced maintenance queues, since at the same time another squadron or detachment on the same ship increased its number of aircraft.

The response concerning potential sorties versus ready hours was most illuminating. If all airwing inputs were held constant, the only apparent way to get more sorties was to increase the length of the flying day, with a resulting reduction of aircraft ready hours. The problem was seen as: Can the ship support a longer flying day? If so, at what cost (more personnel to run catapults and tower, reduction of safety, etc.)? During the Vietnam war, shore-based tactical air, such as at Da Nang during the Khe Sanh or Tet crisis, had surged to an increased sortie rate by stretching out the flying day. These potential airwing economies of scale may be offset by the ship or airfield diseconomies of scale. In addition, a tactical air sortie late or early in the day may not be as effective as a prime time sortie.

Concerning the problem of what is the best attack airwing mix, the relative utility of an attack F-4, A-7, A-6, or A-4 sortie is needed to clarify the situation.

Most experts interviewed considered an effective sortie as some function of payload radius, delivery accuracy, loiter time, and survivability. The differences between current airwing aircraft payloads and delivery accuracy are sizable. General agreement could not be met concerning the relative utility (λ) of different aircraft (depends on the target defenses, type of weather conditions, etc.), since for several situations the relative attack aircraft scale, compared to an A-7 aircraft, appears to be as follows:

Aircraft	(1) Payload radius	(2) Delivery accuracy	(3) Survivability	(4) (1) x (2) x (3) λ Scale	(5) $\overline{(1) + (2) + (3)/3}$ $\bar{\lambda}$
A-7	1.00	1.00	1.00	1.00	1.00
A-6	1.90	.45	.90	.77	1.08
A-4	.40	.60	.80	.19	.60
F-4	.90	.85	1.15	.88	.97

Column (1) values above are on an interval scale. Columns (2) and (3) values tend to be ordinal values. Therefore the λ scale is a biased combination of these values. The $\overline{\lambda}$ is also biased but uses different weighted values. Both λ and $\overline{\lambda}$ will be used later to demonstrate a methodology that should indicate the direction of the best attack airwing mix under uncertainty, concerning the relative value of different types of aircraft sorties.

Finally, two general comments were made aboard the ships visited. One, that the relative importance between spare parts and support equipment varies with the type of aircraft; the payoff of an additional increment of spare parts for the A-7 may be larger than for the A-6 aircraft. Two, there are several ship needs that are not receiving sufficient attention, such as improved internal communications, improved handling of materials, and more ship control of shore training of maintenance personnel (including travel and per diem funds).

SECTION V

THE COMPUTER PROGRAM AND AN ILLUSTRATION OF RESULTS

This section describes the CDC 3800 computer used and the program written to handle the observed data in relation to the Cobb-Douglas type production function model. Based on this program a search of the algorithm is traced through the feasible region toward the boundary. The application of this methodology is then described using the F-4 aircraft for illustrative purposes. And finally, the general case objective function for a total airwing is described.

GENERAL DESCRIPTION

The abstract model of the postulated Cobb-Doublas production function was used as a point of departure in forming the block diagram, figure 4. The program, appendix D, was written for use on the Control Data Corporation (CDC) 3800 computer system in FORTRAN IV language. The 3800 is a solid-state stored program, general-purpose, digital computer. With its large storage capacity and fast data transmissions and computation speeds, it is very suitable for solving large-scale problems. A typical running time for the observations of one type of aircraft, three starting points, and 100 iterations is slightly less than 2-1/2 minutes. Diagnostic runs to test for near convergence to a "best" solution took about 24 minutes each.

SPECIFIC DESCRIPTION

The demonstration or testing of the applicability of the developed methodology starts with the real world of 3M observations in order to determine the elasticities of the inputs. The 3M observations have been edited for completeness, and the units have been standardized (labor months instead of labor hours). Finally, the observations were recorded on magnetic tape. The format is in lengths of 120/1920 characters, with the data in 26 rows. For the purposes of this section only eight rows of information are used. There are two rows for identification (TAGS), two rows for output (U_1, U_2), and four rows for input (W_1 to W_4). The other rows contain such information as hours awaiting parts, labor overtime, etc: the latter information was used in developing the relationships reported in the preceding section.

For the actual processing of the data, see the block diagram from START to step 36 in figure 4. The specific identifications (TAGS) of observations were determined manually. If the number of observations (NOOB) is fewer than 30, then additional observations are supplied manually. The number of iterations (NOIT) was usually 30, but the NOIT has been extended to 100. Sensitivity of the starting points has been tested over a wide range - from .05, .95 to .95, .05. In general, three starting points are used, .25, .75; .50, .50; and .75, .25. The starting point of .50, .50 appeared to be the one that permitted the readhing of a near absolute minima in the fewest iterations.

With the 30 observations, an initial starting point, and the initial starting point, and the initial values for W_1, W_2, etc., the first iteration values for $\alpha_0, \alpha_1, \alpha_2, \ldots \alpha_4$ are obtained where we had a minimum value for:

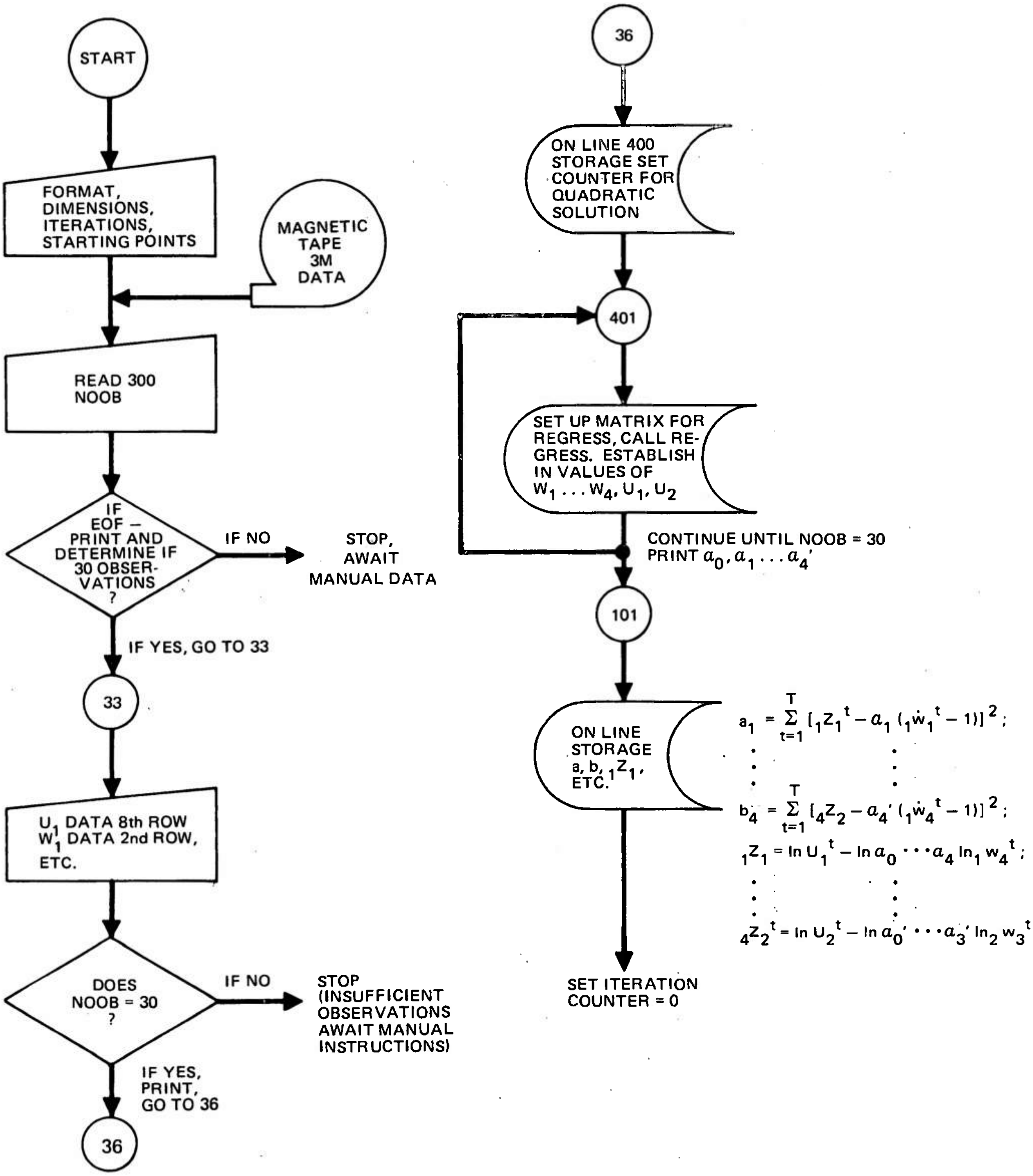


FIG. 4: COMPUTER BLOCK DIAGRAM

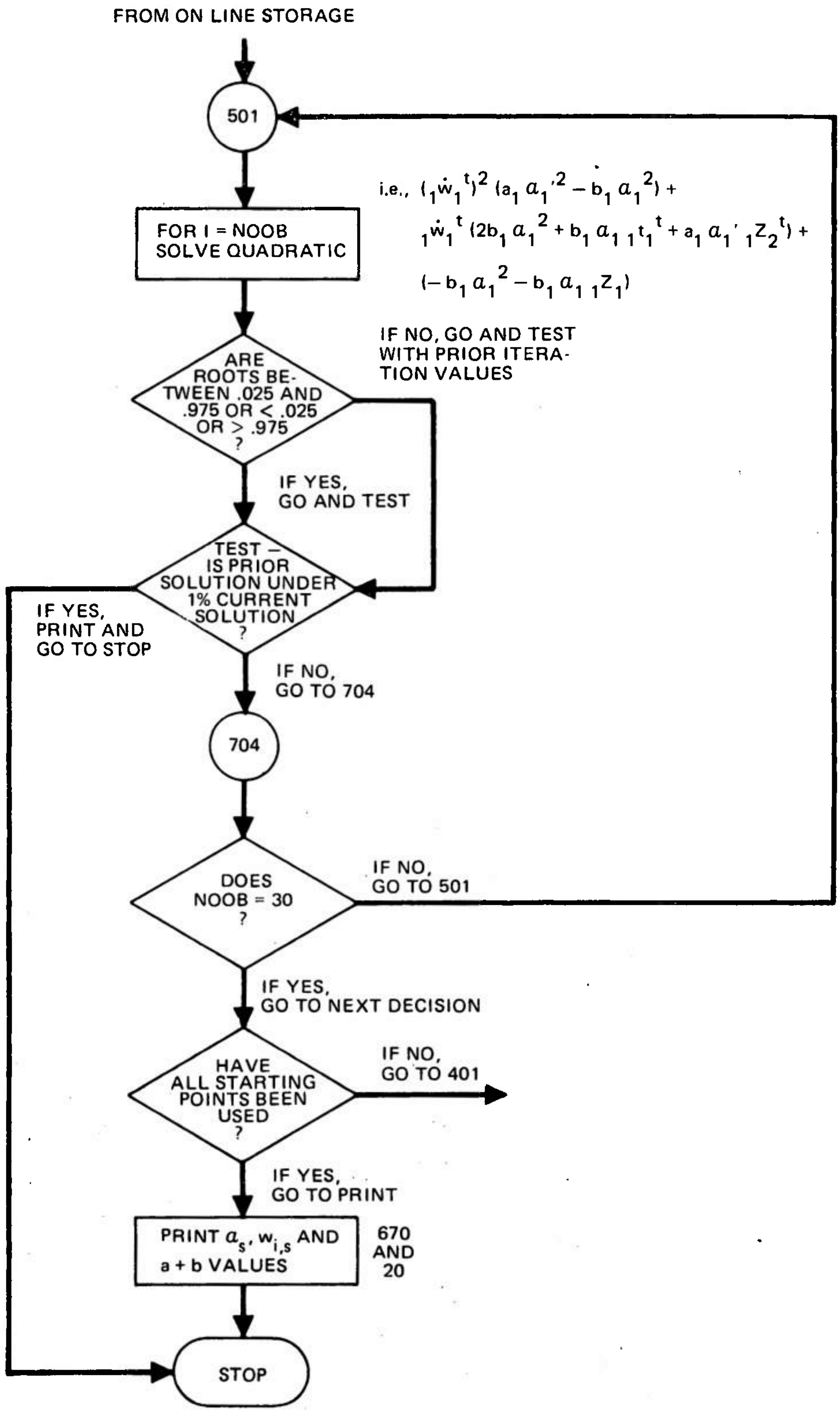


FIG. 4: COMPUTER BLOCK DIAGRAM (Continued)

$$\ell\eta \sum_{t=1}^{T} [\ell\eta U_1^t - \alpha_1 \ell\eta_1 W_1^t \ldots \alpha_4 \ell\eta_1 W_4^t] + \ell\eta \sum_{t=1}^{T} [\ell\eta U_2^t - \ell\eta\alpha_0' - \alpha_1' \ell\eta_2 W_1^t \ldots \alpha_4' \ell\eta_2 W_4^t]^2$$

See REGRESS (Y,X,B,NOOB) of appendix D for the actual subroutine to handle this regression expression.

Then using this first set of values for $\alpha_0, \alpha_1, \alpha_2, \ldots \alpha_4'$, the updated values for ${}_1W_1^t$, ${}_2W_1^t, \ldots {}_2W_4^t$ were obtained for each of the 30 observations by the quadratic methodology. This methodology is documented in appendix D, starting at step 101 through 503. At this point the first iteration has ended.

The second iteration starts with taking the updated values for ${}_1W_1^t \ldots {}_2W_4^t$ and using them in the subroutine REGRESS to obtain revised values for $\alpha_0, \alpha_1, \ldots \alpha_4$. Next the quadratic program is used to revise the values for ${}_1W_1^t \ldots {}_2W_4^t$ and so on until the required number of iterations (NOIT) have been reached or the values for α_j's stabilize from iteration to iteration.

At this point, it should be noted that a convergence problem exists when using the real world (3M) data. As the first local minimum is approached (about the sixth iteration), the quadratic methodology continues to indicate or show the direction that will improve the allocation of resources (revised values of ${}_1W_1, {}_2W_1$ etc., such that they continue to satisfy $W_1 = {}_1W_1 + {}_2W_2$ etc.), but this methodology does not appear to give a precise indication of the distance to move in the next iteration.

As the search moves through the feasible region toward the boundary, an increasing number of observations occur where the quadratic solution for one or more of the inputs is of the type ${}_1W_j^t = 0$ or ${}_2W_j^t = 0$. This would mean that all the inputs are just producing one of the outputs, neglecting the other. Under this unrealistic allocation of inputs, the values for a and b (equations 3.11, 3.12) approach α or 0 respectively, and in the vicinity of zero the partial series expansion of $\ell\eta k_1$ has the most bias. As a result of this, the next iteration of the quadratic format tends to become unstable, since now some of the roots have imaginary solutions $(\sqrt{-1})$ due to this bias.

Where imaginary roots have occurred, the values of the prior iteration of ${}_1W_i$ are used in order to continue the search process. At this point the values for $\ell\eta a + \ell\eta b$ cease to indicate an approach to a local solution, and usually a large increase in the value of $\ell\eta a + \ell\eta b$ occurs. One or two iterations later, and on a zigzag vector away from the global solution, the iteration process starts again to converge to a new local. Figure 5 is a plot of values of (a+b) versus iterations. In the case of the F-4 aircraft data, the first local minimum was reached at about the 12th iteration. After that, near similar locals

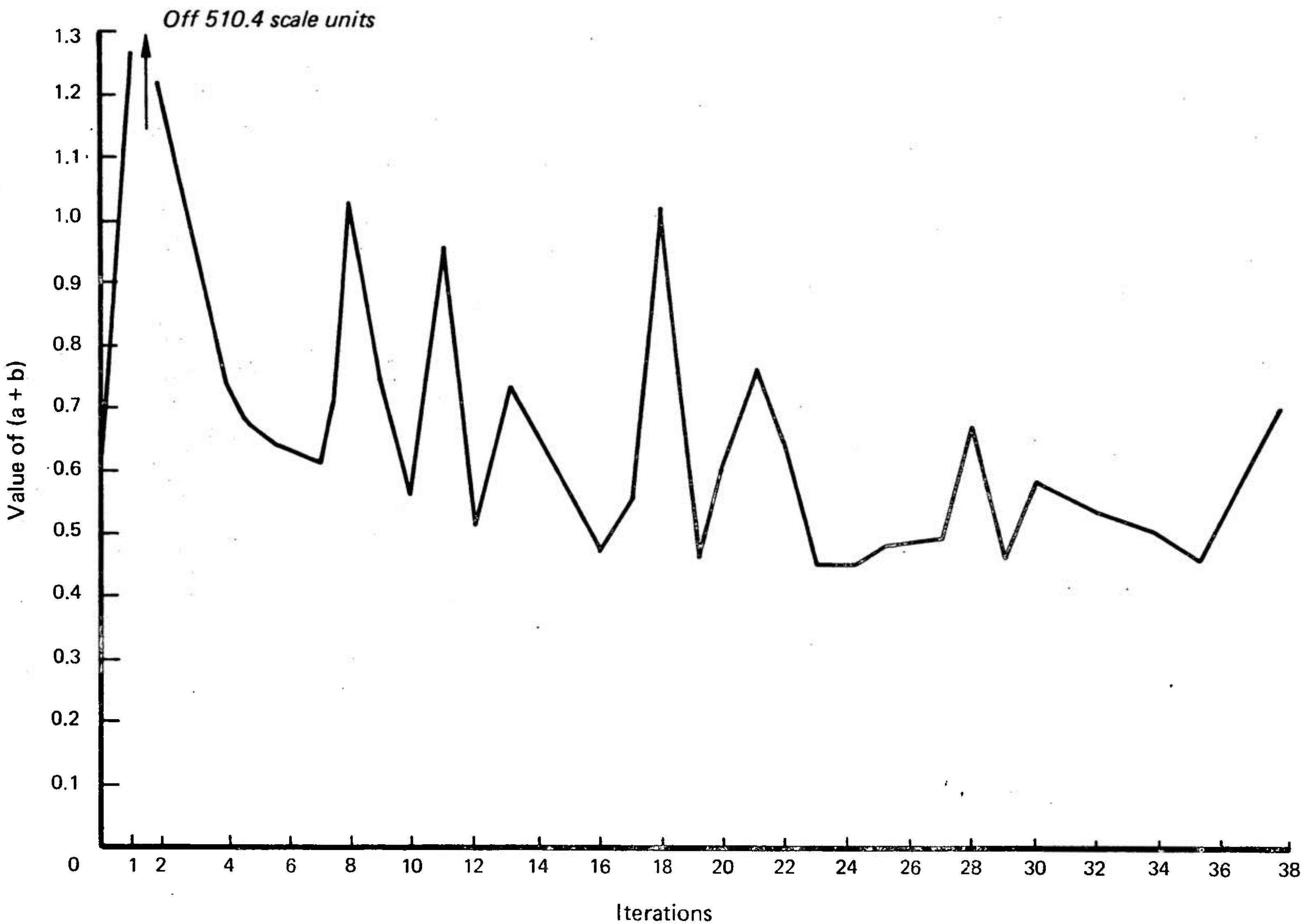


FIG. 5: PLOT OF ITERATIONS AND VALUES

were obtained in the 16th, 23rd, 29th, and 35th iterations. The F-4 data has been run up to 100 iterations without any noticeable change in this pattern. Cycling may be occurring after the 25th iteration.

Different starting points have been used and usually they result in obtaining approximately the same local solution. Extreme starting points (i.e., .05, .09) have resulted in convergence to unwanted stationary points that have values that are physically impossible or which do not provide the best minimum value for (3.8). Attempts to use a better derivative value for (3.10) (via a grid of points), rather than the partial series expansion, have resulted in a large number of unwanted stationary points. None of the stationary points tested gave a better minimum value for (3.8) than that which was obtained by the use of the partial series expansion model shown in figure 4. For diagnostic purposes, printouts were made of all the interim values for W_j^t, iteration by iteration. A grid and manual plot was then made of ${}_1\dot{W}_j^t$ (recall, ${}_1\dot{W}_j^t = \frac{{}_1W_j^t}{W_j^t}$), for its domain of 0 to 1, in order to determine where an equation of the type

$$\ell n \sum_{t=1}^{T} (\tilde{U}_1^t - \alpha_1 \ell n k_1^t)^2 + \ell n \sum_{t=1}^{T} (\tilde{U}_2^t - \alpha_1 \ell n \bar{k}_1^t)^2$$

reaches a minimum.

As expected, the minimum values were not at the limits of ${}_1\dot{W}_j^t$ (0 or 1) but rather from 15 percent to about 60 percent of the distance from 0 to 1. Field trips confirmed that you never have all of the resources of one type, say labor, going to one output (U_1) and none of the same type of resource going to the other output (U_2). Unfortunately, some reported 3M data leads to just such an incorrect allocation of assets. An example of this is the 8th iteration (figure 5). At this point we have an increasing number of imaginary roots and instability for one or two iterations.

Figure 6 illustrates a major portion of the localization problem. At first, each iteration gives us improved values for $\alpha_0, \alpha_1, \ldots \alpha_4$ and ${}_1W_j^t$. Then, when the mean value of the W_j^t observations $(\overline{W_j})$ is estimated to be near the solution boundary, some of the outlying observations have such W_j^t values that they probably "pierce" the hull of the solution area. Once the search has returned to within the feasible region, the quadratic methodology starts on a different vector for the boundary region.

Considering the wide range of different starting points used and the fact that the search was continued for up to 100 iterations, it is reasonable to assume that the average lower value shown in figure 5 (iterations 16, 23, 29 and 35) is a near convergence to an absolute minimum.

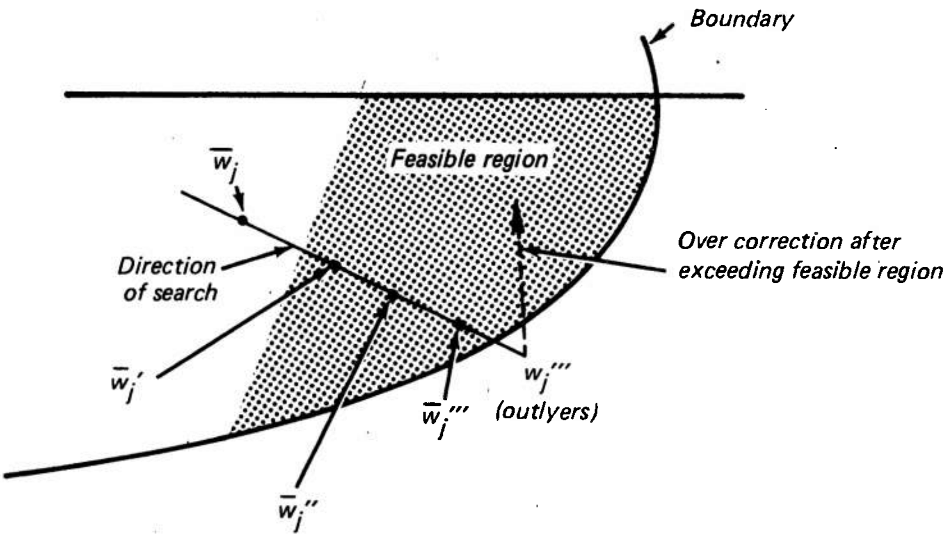


FIG. 6: A SECTION OF THE LOCALIZATION PROBLEM

In this F-4 aircraft case, the five low (a+b) value iterations give the following results:

Iteration	Sortie elasticities α_1	α_2	α_3	α_4	Ready-hour elasticities α'_1	α'_2	α'_3	α'_4
12	.028	.022	.010	.025	1.23	-	.181	.210
16	.031	.010	.060	.015	1.12	-	.279	.110
23	.035	.032	.015	.015	1.13	-	.564	.075
29	.045	.037	.025	.035	.92	-	.591	.120
35	.084	.064	.015	.030	1.05	-	.518	.080
$\bar{\alpha}_j$	.044	.033	.025	.025	1.09	-	.420	.120

Grouping the coefficients together by inputs, we have:

	W_1	W_2	W_3	W_4	
Sorties	.044	.033	.025	.025	Σ .127
Ready hours	1.090	--	.420	.120	Σ 1.630

Recall the objective function (3.7) is:

$$\text{Maximize } U_1 = \alpha_0 W_1^{\alpha_1} W_2^{\alpha_2} W_3^{\alpha_3} W_4^{\alpha_4} + |f_2| \, \Delta\alpha'_0 W_1^{\alpha'_1} W_2^{\alpha'_2} W_3^{\alpha'_3} W_4^{\alpha'_4} \; ; \; X_k \, , \, Y_\ell \; .$$

For the basic policy case (X_1, Y_1), which is cyclic flying operations for 12 hours per day and the fixing of all discrepancies as occuring, we have an average ready hour residual of 400 hours/aircraft/month, or about 56 percent availability. The value for f_2 is $-.546$ when $RH = 400$; thus, for the F-4 the objective function becomes:

$$U_1 = 187.52({}_1W_1)^{.044}({}_1W_2)^{.033}({}_1W_3)^{.025}({}_1W_4)^{.025} +$$

$$|.546| \, \Delta(3.21)({}_2W_1)^{1.09}({}_2W_2)^{.0}({}_2W_3)^{.420}({}_2W_3)^{.120}$$

and at $RH = 400$, the $R^2 = .926$;

at $RH = 300$, the $R^2 = .884$;

and at $RH = 500$, the $R^2 = .913$.

Analysis of residuals by the Theil metholology[1] gives the following decomposition, when comparing estimates and actuals:

$U^m \sim$ .0, bias proportion

$U^s =$ .263, variance proportion

$U^c =$ .737, covariance proportion.

From this it appears the function (3.7) and the methodology for obtaining the coefficients of elasticities is unbiased and reasonably specified. The remaining covariance (U^c) is a variable that cannot be exactly estimated. Due to the limitations of the reported observations, it appears that not much can be done about the covariance proportion of this error.[2]

GENERAL APPLICATION OF THE SPECIFIC DESCRIPTION

Recall the objective function for the F-4 is:

$$U_1 = 187.52({}_1W_1)^{.044}({}_1W_2)^{.033}({}_1W_3)^{.025}({}_1W_4)^{.025} +$$

$$|.546|(3.21)({}_2W_1)^{1.09}({}_2W_2)^{0}({}_2W_3)^{.420}({}_2W_4)^{.120} \text{ at}$$

$$RH = 400 \; .$$

Using the average allocation of resources in our observations, this becomes:

$$U_1 = 187.52(3)^{.044}(95)^{.033}(50)^{.025}(18)^{.025}$$
$$+ |.546|(3.21(9)^{1.09}(55)^{0}(37)^{.420}(61)^{.120} - 262.66)$$
$$= 270.95 \text{ sorties/month} \tag{5.1}$$

If all resources are increased by 10 percent, holding RH = 400 constant, we have:

$$U_1 = 187.52\,(3.3)^{.044}(104.5)^{.033}(55)^{.025}(19.8)^{.025} +$$
$$|.546|(3.21)(9.9)^{1.09}(60.5)^{0}(40.7)^{.420}(67.1)^{.120} - (262.66)\,|.546|$$
$$= 274.415 + 24.089$$
$$= 298.504 \text{ or an increase of } 27.552 \text{ sorties, or } 10.168 \text{ percent.}$$

Thus, for a 10 percent increase in inputs we have about a 10 percent change (10.168 percent in outputs, or the summation of the elasticities $\alpha_1 + \alpha_2 + \alpha_3 + \alpha_4 + f_2\,(\alpha'_1 + \alpha'_2 + \alpha'_3 + \alpha'_4) \approx 1$. For this specified policy (RH = 400) we have almost constant returns to scale as resources are increased.

The increased output (+27.552 sorties) was a result of a 10 percent increase in inputs, weighted for the individual coefficients of elasticity. Specifically, the increased sorties were a result of:

+ 17.319 sorties - due to an increase of 1.2 aircraft (W_1)

+ 0.894 sorties - due to an increase of 14.5 men (W_2)

+ 6.886 sorties - due to an increase of 8.7 units support (W_3)

+ 2.454 sorties - due to an increase of 7.9 units spares (W_4)

Σ 27.553

Comparing the above marginal physical products to their price or cost gives a ranking of the outputs in relation to their basic cost.[3] The ranking is:

$$W_3 \text{ (support)} = \frac{6.886}{(8.7)(.666)} = 1.188 \text{ sorties/thousand dollars (K)}$$

$$W_4 \text{ (spares)} = \frac{2.454}{(7.9)(.869)} = 0.357 \text{ sorties/K}$$

$$W_1 \text{ (aircraft)} = \frac{17.319}{(1.2)(46.2)} = 0.312 \text{ sorties/K}$$

$$W_2 \text{ (men)} = \frac{0.894}{(14.5)(.466)} = 0.132 \text{ sorties/K}$$

Recall that the basic allocation of inputs in the observed F-4 squadrons (5.1) generates a potential of 270.95 sorties/month. This basic case had the following monthly budget associated with this potential to generate sorties:

		Percentage of total
Aircraft	12 x 46.2 = 554.40	74.1
Men	145 x .466 = 67.57	9.0
Support	87 x .666 = 57.94	7.7
Spares	79 x .869 = 68.65	9.2
	\$748.6 K/month[4]	100.0 percent
or	\$ 8.983 M/year	

Then, if the constraints of the F-4 objective function (5.1) are:

$$9 \le W_1 \le 14$$

$$140 \le W_2 \le 165$$

$$70 \le W_3 \le 125$$

$$70 \le W_4 \le 150$$

$$C_1W_1 + C_2W_2 + C_3W_3 + C_4W_4 \le 748.6 \text{ K/month} \;.$$

A best mix equal cost squadron is:

$$\begin{aligned} U_1 &= 187.53(2.83)^{.044}(88.7)^{.033}(71.8)^{.025}(20.0)^{.025} + \\ &\quad |.546| \, [3.211(8.50)^{1.09}(51.3)^{0}(53.2^{.420}(70.5)^{.120} - 262.66] \\ &= 272.96 + 16.38 \\ &= 289.34 \text{ sorties} \end{aligned} \tag{5.2}$$

This is an increase of 18.39 sorties or 6.8 percent over the base case (289.34 - 270.95). This increase occurs with the following shifts in basic inputs:

- \- .67 aircraft
- \- 10.0 men
- \+ 38.0 units of support
- \+ 11.5 units of spare parts

For the constraints of the situation, we have suboptimized the F-4 mix of inputs. At a higher level the problem is to determine the best overall airwing mix (A-6, A-7, F-4, etc.) to maximize the total carrier output.

If the relative utility (λ) between types of aircraft is:[5]

Aircraft	λ	$\bar{\lambda}$
A-7	1.00	1.00
A-6	.77	1.08
A-4	.19	.60
F-4	.88	.97

then the total objective function becomes:

Maximize

$$(A\text{-}7)\,\alpha_0 W_1^{\alpha_1} \ldots W_4^{\alpha_4} + (.77 \text{ to } 1.08)_{(A\text{-}6)}\,\alpha_0 W_1^{\alpha_1} \ldots W_4^{\alpha_4}$$

$$+ (.88 \text{ to } .97)_{(F\text{-}4)}\ \alpha_0 W_1^{\alpha_1} \ldots W_4^{\alpha_4}\quad , \tag{5.3}$$

subject to the limitations of the upper and lower bounds on the inputs (i.e., total of W_1 [aircraft] $\leq$ total space) and a budget constraint.

However, before an evaluation can be made concerning the usefulness of this general objective function, we must first establish the possible ranges that the elasticities (α_1, $\alpha_2, \ldots \alpha_4$) can have under the specific constraints for each type of aircraft. So far these values have been established only for the F-4 aircraft. The next section will cover the determination of the elasticities for the other aircraft (A-7, A-6, A-4 and E-2). Then, with these elasticities, the problem of the "best" average cost for an attack sortie can be evaluated. If the ratio between average cost per sortie per type of aircraft exceeds the ratio of the relative utility values between the same aircraft, one should probably shift the airwing mix so that it will favor a lower average cost per equivalent sortie.

FOOTNOTES

[1]Henri Theil, Applied Economic Forecasting, Amsterdam: North Holland Publishing, 1966, pp. 29-30.

[2]Interview information from the 3M Headquarters at Mechanicsburg, Pennsylvania alerted me to the sometimes "noisy" 3M data. To date no high R^2 have been obtained from these inputs. Sutton obtained an R^2 of about .814 using one less independent variable and not excluding the "noisy" observations.

[3]See appendix E for cost data for each input for each type of aircraft.

[4]This is peacetime budget related to generating a wartime capability. When the time comes to use this output fully, there will be the additional direct cost of consumables (fuel, oil, etc.) and possibly additional aircraws (if the sortie rate exceeds the planned rate). These consumables and aircrew costs are not directly related to the peacetime allocation decision between the $W_1 \ldots W_4$ inputs. Therefore, consumables can logically be excluded from this phase of the research without affecting the relative tradeoffs between the factors pertaining to potential sorties.

[5]Infra. p. 50.

SECTION VI

THE RESULTS OBTAINED FOR THE F-4, A-7, A-6 AND E-2 AIRCRAFT

This section describes in detail the results obtained from a series of Cobb-Douglas type production functions in relation to the observed data. Based upon these specific aircraft production function relationships, a possible total airwing objective function is then described. Finally, a summary is made of these results to indicate possible trade-offs between types of aircraft.

F-4 RESULTS, GENERAL COMMENTS

The previous section described the development of the F-4 production function. Recall that the elasticities were:

	α_1	α_2	α_3	α_4	
Sorties (U_1)	.044	.033	.025	.025	Σ .107
Ready hours (U_2)	1.090	--	.420	.120	Σ1.630

Three samples in all were drawn from the total observations, each with a different average level of ready hours. For these samples the coefficient of determination (R^2) ranged from .884 to .926. The highest R^2 (.926) occurred at the most frequently observed flying hour policy (RH = 400).

A sequential F-test of the variables for the sortie output was made to determine whether each variable appeared to contribute sufficiently. The worst case was taken (ready hours held constant) to test whether the "small" sortie elasticities warranted elimination of any variables. The F values and the standard F critical values are:

Variable	F	Critical value
W_2	6.974	F(1, 28, 0.95) = 4.20
W_4	6.958	F(2, 27, 0.99) = 5.49
W_3	6.115	F(3, 26, 0.99) = 4.64
W_1	5.062	F(4, 25, 0.99) = 4.18

Since the F values in all cases exceed the critical values, each of these variables is worthwhile. The residuals were analyzed for three types of possible error. No sampling bias error could be detected. The variance proportion of the residual error was about .263, and the covariance proportion was about .737. A plot of residuals indicated some difficulty with outlier observations, particularly from those of squadrons aboard USS KENNEDY. Keypunch errors and errors in the interpretation of reporting requirements probably account for the majority of 3M outliers.

When the observations were arranged in chronological order (July 1968 to October 1970), the Durbin-Watson test gave a statistic of about 2.0. But when the observations were ranked by their relative position within a cruise (all first months, then second month at sea, etc.), a degree of serial correlation was present. The Durbin-Watson statistic was 2.583. This is significant to the five percent level of confidence. This confirms the intuitive opinion that a definite re-learning process takes place within a squadron for each and every cruise. This is probably an administrative expense as a result of the Navy's high personnel turnover rates.

The labor elasticity for ready hours (α_2') tended to be negative. But negative elasticities are meaningless.[1] For the observed range of inputs it is improbable that output would actually decrease with an increase in input. To operate in other than the feasible region ($\alpha_j \geq 0$) fails to follow economic logic. Finally, the degree to which α_2' tended to be negative was not statistically significant. In other words, the hypothesis that this elasticity is zero cannot be rejected. Although this tendency for the wrong sign is a warning of possible incorrect specification, the model is empirically preferred. Of course one does not choose just the equation with the highest R^2. The included inputs (w_1 to w_4) are based on prior research, such as done by Gilster and Sutton, observations and theoretical considerations. The model appears to be reasonably specified and explains most of the variation observed.

Variations in Policy

Recall that in the prior section the summation of the elasticities ($\alpha_1 + \alpha_2 \ldots$, etc.) was about 1.0 for a specified policy level of flying (RH = 400). This means that a 10 percent increase in inputs will lead to about a 10 percent increase in output. However, if the flying policy is changed, what will happen to the output? If the flying hour day is increased until ready hours decrease to RH = 300, then the output will increase as high as:

$$U_1' = 187.52(3.3)^{.044}(104.5)^{.033}(55)^{.025}(19.8)^{.025} + |.600|\left[3.21(9.9)^{1.09}(60.5)^{0}(40.7)^{.420}(67.1)^{.120} - 262.66\right] \tag{6.1}$$

$= 274.415 + 26.471$

$= 300.886$ sorties or an additional increase in output of + .879 percent due to a change in policy. This means that a 10 percent increase in inputs will result in economies of scale (11.047 percent output).

In a similar manner, if the flying day were decreased, holding all inputs constant (with an increase in readiness of RH = 500), the output would decrease to:

$$U_1' = 187.52(3.3)^{.044}(104.5)^{.033}(55)^{.025}(19.8)^{.025} + |.505|\left[3.21(9.9)^{1.09}(60.5)^{0}(40.7)^{.420}(67.1)^{.120} - 262.66\right] \tag{6.2}$$

$= 274.415 + 22.280$

$= 296.695$ sorties or a decrease of .667 percent over the basic output at policy RH = 400. This means for a 10 percent increase in inputs the output increased only 9.502 percent, or diseconomies of scale occurred.

If the maintenance policy is changed to progressive, on-condition, scheduled maintenance, and the deferring of flight discrepancies were possible, for batch processing the labor savings might be 37-1/2 percent for scheduled maintenance and 10 percent in non-scheduled maintenance. This amounts to an overall savings of 16.9 man months (about 10 percent of the labor force) or \$7.875 K/month (16.9 x .466). There will also be an increase in readiness (expert opinion estimates at least a 5 percent increase). Using the marginal physical products quantified earlier, this budget saving could purchase sufficient support equipment for an additional 4.17 sorties or an increase of 1.54 percent in output. The potential increased readiness might give an additional 7.15 sorties or a +2.64 percent of output.

Thus, while variations in policy (maintenance and flying) may not have a dominant effect on output, they increase the output somewhat and change the economies of scale of the production function. Figure 7 illustrates the F-4 production surface for three levels of flying. In this case $U_1U_2 = U_2U_3 = U_3U_4 = U_4U_5$ (constant returns to scale), $U'_1U'_2 > U'_2U'_3 > U'_3U'_4 > U'_4U'_5$ (decreasing returns to scale), and $U''_1U''_2 < U''_2U''_3 < U''_3U''_4 < U''_4U''_5$ (increasing returns to scale).

Figure 7 clearly indicates the different zones of production - U''_1, U''_2, ... U''_5 (economies of scale), U_1, U_2, ... U_5 (constant returns) and U'_1, U'_2, ... U'_5 (diseconomies of scale).

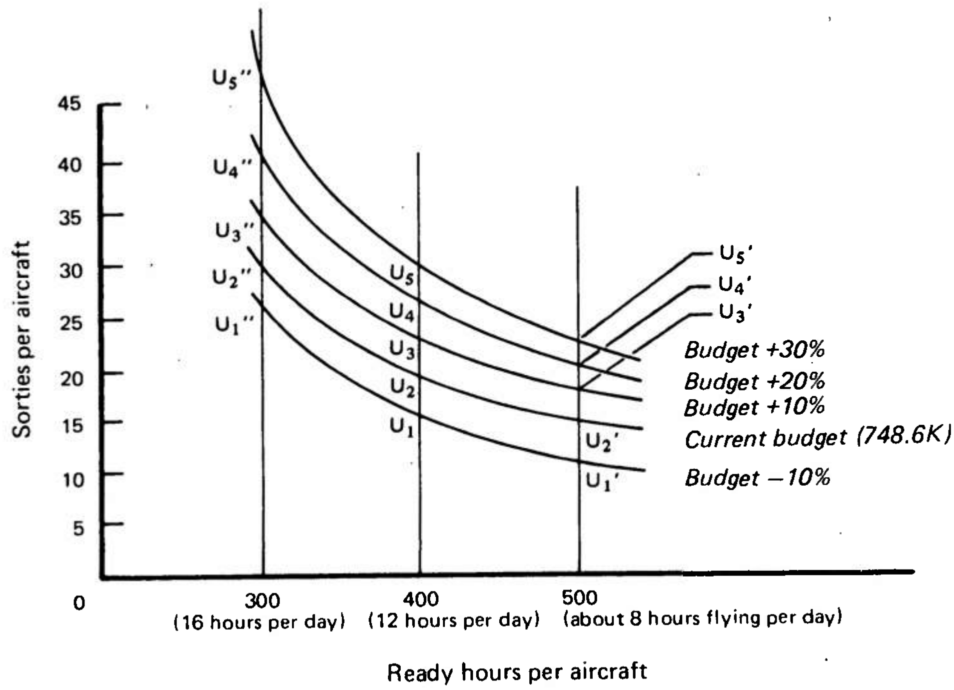


FIG. 7: F-4 PRODUCTION SURFACE

Figure 8 is a composite projection of figure 7 on a plane perpendicular to the X (Ready Hour) axis.

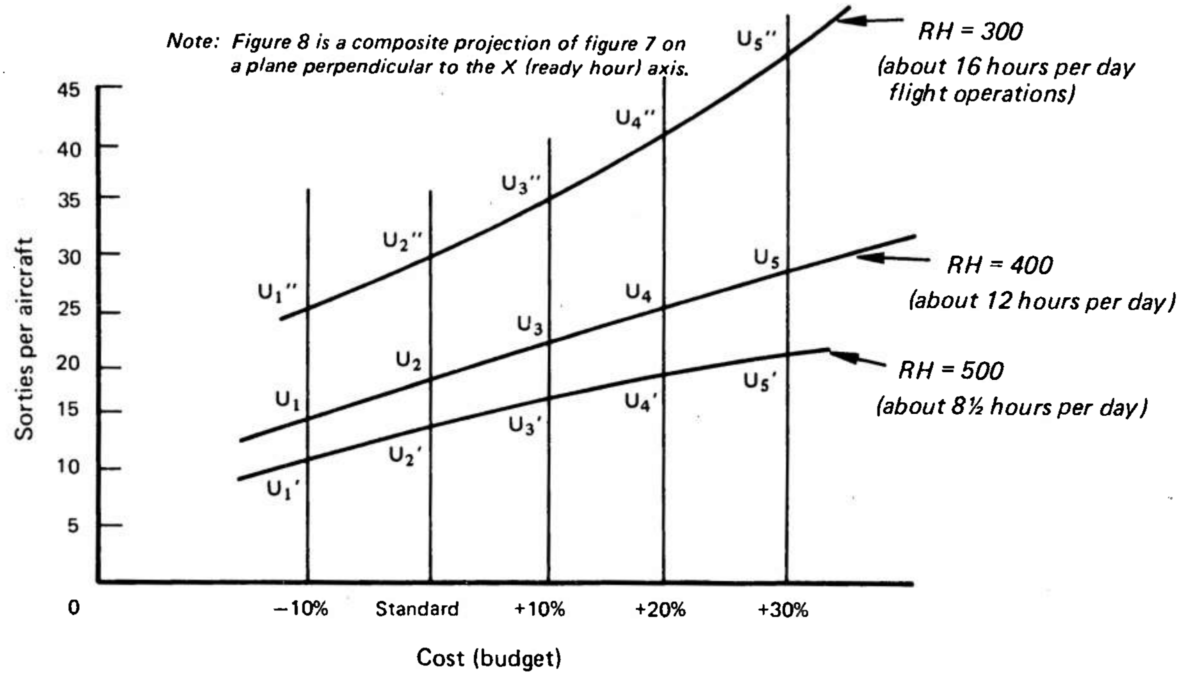


FIG. 8: ECONOMIES OF SCALE

Sensitivity to Cost Changes

Recall from the previous section that a "best mix," equal-cost F-4 squadron had a maximum output of 289.34 sorties at RH = 400. The solution was reached after W_1 - .67, W_2 - 10.0 and increasing W_3 + 38.0 and W_4 + 11.5. This corresponds to increasing the use of resources that have a high ratio of MPP/Cost and decreasing those with a low MPP/Cost.

The range of MPP/Cost for each input is shown below:

TABLE I

Inputs	Units of cost[a] $_1C$	$_2C$	$_3C$	$_4C$	Priority of substitution
Aircraft W_1	.339	.312	.208	.411	2nd or 3rd
Men W_2	.132	.107	.072	--	Last
Support W_3	1.188	1.007	--	--	1st
Spares W_4	.357	.288	--	--	2nd or 3rd

[a] Appendix E.

The W_3 input dominates regardless of the cost variation. The W_4 input is always last. The other two (W_1, W_4) are sensitive to price changes. In this case the relative value of one factor of production can surpass the other depending on which costing decisions are used.

Average versus Marginal Cost

Recall that for a fixed budget of $748.6 K/month the best mix produced an output of 289.34 sorties. The average cost (AC) per sortie is $2.587K. But the marginal cost (MC) to generate additional sorties by the use of each possible input is the reciprocal of the MPP_{W_j}/C_j, which is:

MC_1 = 3.205, using aircraft at ${}_2C_1$ cost

MC_2 = 7.576, using men at ${}_1C_2$ cost

MC_3 = .842, using support equipment at ${}_1C_3$ cost

MC_4 = 2.801, using spare parts at ${}_1C_4$ cost.

Since MC for W_3 < AC, it would be logical to increase this input until MC = AC, provided $W_3 \le 125$ and if the budget ($748.6 K/month) can be exceeded. Unfortunately this factor of production is restricted already by the upper constraint. No additional support equipment can be added, as the problem has been defined. Therefore the AC of $2.587K is the best average cost obtainable within the defined constraints.

Note, however, that MC_4 is only a small (percent) amount above the best AC and the constraints of the situation ($W_4 \le 150$) permits an additional 59.5 units of spare parts. This means a 16.06 additional sorties or 5.6 percent can be purchased with only a minimal increase in average cost if additional sorties are required. If additional output is required, this may be an attractive alternative, say compared to new ship/airwing construction.

A-7 RESULTS, GENERAL COMMENTS

The regression analysis of the 30 observations resulted in similar search patterns to that of the F-4. The first near convergence to a solution was reached in the 11th iteration. An average of the five lowest (a+b) values gave the following results:

	α_1	α_2	α_3	α_4	
Sorties (U_1)	.040	--	.043	.032	Σ.115
Ready hours (U_2)	.933	.050	--	.381	Σ1.364

This aircraft sample of observations had the highest coefficient of determination, compared to the other types of aircraft used in this study. The R^2 was .9513. The analysis of the residuals indicated the error of central tendency or sampling bias was about zero. The variance proportion of the error was .421 and the covariance proportion was .579. Again the Cobb-Douglas type equation appears unbiased and reasonably specified, however, the inherent variations within the 3M data does not permit us to estimate exactly the relationship between inputs and outputs.

When the observations were arranged in the order of their occurrence within a cruise (all first month, then second months, etc.), the Durbin-Watson statistic was 2.2490. Again a positive degree of serial correction was present but not at a level that is statistically significant.

The elasticities of labor (α_2), for the output of sorties (U_1), and support (α'_3), for the output of ready hours (U_2), tended to be negative. However, the degree was not sufficient to be statistically significant, and therefore the hypothesis that the negative elasticity is zero cannot be rejected. In addition, there is the possibility that the 3M data involving support equipment is more a measure of assets present than assets actually used. Research by others report similar problems associated with the measurement of the input of capital services.[2]

Variations in Policy

The level of readiness for the average observation of this sample was 64 percent at RH = 460. At this level of readiness a 10 percent increase in inputs resulted in an increase of output of 10.101 percent or 30.63 sorties. Thus, the summation of the elasticities $\alpha_1 + \alpha_2 + \alpha_3 + \alpha_4 + f_2 (\alpha'_1 + \alpha'_2 + \alpha'_3 + \alpha'_4) \approx 1$. For this specific policy (RH = 460), we have near constant returns to scale as the resources are increased. If we decrease the readiness policy to RH = 360, the slope or rate of trade-off between potential sorties and ready hours increases from .646 to .678. If we increase the readiness policy to RH = 560, the slope decreases to .614. At RH = 360 the economies of scale changed from being about constant to 1.059, indicating a tendency to develop some economies of scale. At RH = 560 there is a reverse tendency toward some diseconomies of scale.

Variations in the maintenance policy (on-condition maintenance and deferred discrepancies are possible) can save about 10 percent (13.3 men) on labor. If this \$5.89 K/month wage saving is used to purchase additional spare parts (W_4), we can have an increase of 8.596 sorties or +2.87 percent over the base case. There would be also an accompanying increase in readiness.

Variations in the flying policy (longer or shorter flying day) change the output, but they do not appear to change it significantly. Under constant budget conditions, variations in the maintenance policy can increase the output by at least 8.596 sorties or + 2.8 percent.

Sensitivity to Cost Changes

In the average A-7 squadron we had the following Cobb-Douglas type relationship:

$$\begin{aligned} U_1 &= 226.22(5.7)^{.040}(40)^{0}(30)^{.043}(38)^{.032} \\ &+ |.646| \, [9.525(7)^{.933}(93.6)^{.050}(57)^{0}(41)^{.381} - 302.6] \\ &= 299.06 \text{ sorties;} \end{aligned} \tag{6.3}$$

with a 10 percent increase in inputs, the output increased to 326.39 (increase of 30.63 sorties or 10.1 percent). Specifically the increased sorties were a result of:

+ 19.779 sorties - due to an increase of 1.2 aircraft (W_1)

+ .984 sorties - due to an increase of 13.3 men (W_2)

+ 1.323 sorties - due to an increase of 8.7 units support (W_3)

+ 8.551 sorties - due to an increase of 7.8 units of spares (W_4).

Comparing these marginal physical products to their costs indicates again which factors of production should be substituted for another to achieve or approach the lowest cost combination for a fixed output. If the relative ranking between $MPP_{(\text{input A})}/Cost_A$ to $MPP_{(\text{input B})}/Cost_B$ ..., etc., changes with different prices of the same input, then the direction of substitution will shift. If this happens, it can be concluded that the priority between inputs is sensitive to shifts in cost.

Specifically the MPP/Cost for each input is:

TABLE II

Inputs	Units of cost $_1C$	$_2C$	$_3C$	$_4C$	Priority of substitution
Aircraft W_1	.609	.562	.374	.739	2nd
Men W_2	.171	.136	.092	--	4th
Support W_3	.219	.187	--	--	3rd
Spares W_4	1.495	1.211	--	--	1st

Since the MPP/C for W_4 (1.211 to 1.495) always surpasses all the other factors of production, W_4 should have priority and be substituted for some of the other factors. For decision purposes the relative value of W_4 is insensitive to cost. In a similar fashion, W_1 (aircraft) is always next (.394) to (.739), then W_3, and finally W_2. All of these inputs are insensitive to variations in cost within the ranges under consideration.

Average Versus Marginal Cost

Recall that the average observed A-7 squadron production function (6.3) was:

$$U_1 = 226.22(5.7)^{.040}(40)^{0}(30)^{.043}(38)^{.032}$$

$$+ |.646| [9.525(7)^{.933}(93.6)^{.050}(57)^{0}(41)^{.381} - 302.6]$$

$$= 299.06 \text{ sorties when the RH policy} = 460 \quad .$$

If we maximize the output for the same ready hour and maintenance policy level, subject to the following constraints,

$$10 \le W_1 \le 14$$

$$120 \le W_2 \le 150$$

$$70 \le W_3 \le 125$$

$$70 \le W_4 \le 150,$$

$$C_1W_1 + C_2W_2 + C_3W_3 + C_4W_4 \le \$549.65 \text{ K/month.}$$

We have,

$$U_1' = 226.22(5.35)^{.040}(36)^{0}(24)^{.043}(56)^{.032}$$

$$+ |.646| [9.525(7)^{.933}(84)^{.050}(46)^{0}(60)^{.381} - 302.6] \qquad (6.4)$$

$$= 327.89 \text{ sorties, or an increase of } 28.83 \text{ sorties or } +9.6 \text{ percent.}$$

This "best" mix occurs with the following shifts in inputs:

-.35 aircraft

-12.6 men

-17 units support

+37.1 units spare parts

This "best" mix has an average cost (AC) of 1.670K, But the marginal cost (MC) to generate additional sorties by the use of each possible input is the reciprocal of the MPP_{W_j}/C_j, which is:

MC_1 = 1.779 using aircraft at ${}_2C_1$ cost

MC_2 = 5.848 using men at ${}_1C_2$ cost

MC_3 = 4.566 using support equipment at ${}_1C_3$ cost

MC_4 = 0.685 using spare parts at ${}_1C_4$ cost

Since MC for W_4 < AC, it would be logical to increase this input until MC = AC, provided $W_4 \leq 150$ and the budget ($549.65 K/month) can be exceeded. If W_4 were increased to its limit (+34 units), the output would increase to 352.8 sorties (+ 24.71 sorties or +7.5 percent) at an additional cost of $26.13 K/month. The average cost (AC) would decrease to 1.630, which is the best AC obtainable for the constraints of this problem.

A-4 RESULTS, GENERAL COMMENTS

The regression analysis of these observations continued to give us about the same search pattern as the values converged to a "best" local solution. Specifically, we have:

	α_1	α_2	α_3	α_4	
Sorties (U_1)	.034	.025	.048	--	Σ .107
Ready hours (U_2)	.865	--	--	.238	Σ1.103

The coefficient of determination (R^2) was .9509. Analyses of the residuals indicated the sampling error of central tendency was about zero, the proportion of variance error was about .158, and the proportion of covariance error was .842. The Durbin-Watson statistic for the observations (when ranked in their order of occurrence within a cruise) was + 2.220. Again a positive degree of serial correlation is present but not at a level which is statistically significant.

When the elasticities of inputs tended to be negative, the degree was not statistically significant. The problem of measurement of the capital services inputs again occurs. The A-4 is the oldest aircraft used in this study. Any management support problems associated with this aircraft probably were solved some time ago. Thus, the 3M inputs which contained only selected support equipment have few items pertaining to this aircraft. The elasticities associated with W_3 are not considered too reliable.

Variations in Policy

The level of readiness for the average observations of this sample was 69.4 percent at RH = 500. At this level of readiness, a 10 percent increase in inputs results in an increase in outputs of 10.170 percent or 33.5 sorties. Thus, the summation of the elasticities ($\alpha_1 + \alpha_2, \ldots \alpha'_4$) is $\approx$ 1.0, and we have near constant returns to scale. Unfortunately, there is insufficient information to predict how the slope, or the trade-off of ready hours versus potential sorties, will shift with a change in readiness or length of flying hours. However, it is expected that any increase in the flying day would lead to at least some economies of scale.

As with the other types of aircraft, variations in the maintenance policy can save up to 10 percent on total labor (8.6 men). If this $3.603 K/month wage savings is used to purchase additional spare parts (W_4), we can have an increase of 6.781 sorties or + 2.059 percent increase in output.

Thus, variations in the flying policy do not appear to have a noticeable effect on output. Under a fixed budget, variations in the maintenance policy can at best increase the output by about 2.06 percent.

Sensitivity to Cost Changes

In the average A-4 squadron observed, we had the following Cobb-Douglas type relationship:

$$\begin{aligned} U_1 &= 255.8(4.4)^{.034}(20)^{.025}(14)^{.048}(20)^{0} \\ &\quad + |.797|\,[17.493(9.2)^{.865}(66)^{0}(70)^{0}(69)^{.238} - 326.6] \qquad (6.5) \\ &= 329.2 \text{ sorties} \end{aligned}$$

With a 10 percent increase in inputs, the output increased to 362.7 sorties (increase of 33.5 sorties or 10.17 percent). Specifically the increased sorties were a result of:

+ 24.560 sorties - due to an increase of 1.36 aircraft (W_1)

+ .849 sorties - due to an increase of 8.6 men (W_2)

+ 1.631 sorties - due to an increase of 8.4 units support (W_3)

+ 6.455 sorties - due to an increase of 8.9 units spares (W_4).

Comparing the MPP's to their costs will indicate which factors of production should be substituted for another. If the relative ranking between $MPP_{(\text{input } A)}$/Cost of A to $MPP_{(\text{input } B)}$/Cost of B etc., changes with the different prices of each input, then the direction of substitution will shift, and the priority of substitution of inputs can be considered sensitive to shifts in price.

Specifically the range of MPP/Cost for each input is:

TABLE III

Inputs	Units of cost $_1C$	$_2C$	$_3C$	$_4C$	Priority of substitution
Aircraft W_1	2.052	1.629	1.086	2.851	1st or 2nd
Men W_2	.253	.199	.136	--	4th
Support W_3	.606	.478	--	--	3rd
Spares W_4	1.882	1.507	--	--	1st or 2nd

Since the relative value of MPP_{W_1}/C_1 and MPP_{W_4}/C_4 changes with the various costs considered, then the inputs of aircraft and spare parts are sensitive to costs. On the other hand, the inputs of support (W_3) and men (W_2) are insensitive for the ranges under consideration.

Average Versus Marginal Cost

Recall that the average observed A-4 squadron production function (6.5) was:

$$U_1 = 255.8(4.4)^{.034}(20)^{.025}(14)^{.048}(20)^{0}$$
$$+ |.797| [17.493(9.2)^{.865}(66)^{0}(70)^{0}(69)^{.238} - 326.6]$$
$$= 329.2 \text{ sorties when RH} = 500.$$

If, for the above, we hold constant both the budget (\$245.9 K/month) and the policy (RH = 500), subject to the following constraints,

$$10 \le W_1 \le 15$$
$$75 \le W_2 \le 105$$
$$70 \le W_3 \le 125$$
$$70 \le W_4 \le 150,$$

and if we maximize output using the basic cost inputs, we have

$$U'_1 = 255.8(4.4)^{.034}(17.2)^{.025}(11.4)^{.048}(25)^{0}$$
$$+ |.797| [17.493(9.2)^{.865}(58)^{0}(59)^{0}(87.6)^{.238} - 326.2] \qquad (6.6)$$
$$= 337.92 \text{ sorties or an increase of only } 8.72 \text{ sorties or } +2.65 \text{ percent.}$$

This "best" mix was obtained with the following shifts in inputs:

- 11.8 men
- 13.5 units of support
+ 23.6 units of spare parts.

This best mix had an average cost (AC) per sorties of .728. But the basic marginal cost (MC) to generate additional sorties by the use of each input is the reciprocal of the MPP_{W_j}/C_j, which is:

$$MC_1 = 0.614$$
$$MC_2 = 3.952$$
$$MC_3 = 1.650$$
$$MC_4 = 0.531$$

Since the MC for W_1 and W_4 are < AC, it would be logical to increase the inputs of these two factors until MC = AC, provided $W_1 \le 15$ and $W_4 \le 150$ and that the budget of

\$245.9 K/month can be exceeded. In this case we should increase W_1 to 15 and W_4 to 150 with a revised budget of \$275.8 K/month.

With these increased inputs (W_1 + 1.4 aircraft, W_4 + 37.4 units) equation (6.6) now has an output of 388.71 sorties. This is an increase of 50.79 sorties of + 15.03 percent. The average cost has now decreased to .709. This is the "best" A-4 average cost obtainable within the constraints of the problem.

It should be noted that the A-4 best "mix" had an improvement in output of only 2.65 percent, compared to the average improvement in output for the other aircraft of 6.81 percent. In addition the MC for the A-4 are closer to AC (76 percent) than in the case of the other aircraft (40 percent). This indicates that the current Navy institutional review process, given time, makes allocation decisions that have a net effect of approaching the optimum range where > MC → AC. However the methodology that is the heart of this study should offer a way to speed up the "best" allocation decisions for new aircraft.

A-6 RESULTS, GENERAL COMMENTS

The regression analysis with its iterative search and near convergence to a "best" solution was like that of the other types of aircraft. Specifically, we have:

	α_1	α_2	α_3	α_4	
Sorties (U_1)	.035	.050	--	.019	Σ .104
Ready hours (U_2)	1.075	.041	.109	.152	Σ 1.377

The coefficient of determination (R^2) was .9188. Analysis of the residuals indicated little or no sampling bias, the proportion of variance error was about .469 and the proportion of covariance error was .531. The Durbin-Watson statistic for the observations (when ranked in their order of their occurrence within a cruise) was + 2.551. The positive degree of serial correlation is significant to the 5 percent level of confidence.

The elasticity of support (α_3) tended to be negative for the output of sorties, but not to a degree considered statistically significant. Again we have a limitation when measuring capital services (support equipment) available versus the capital services (support equipment) actually used.

Variations in Policy

The level of readiness for the average observations of this sample was about 69.4 percent at RH = 500. At this level, a 10 percent increase in inputs resulted in an increase in output of only 21.52 sorties or 8.207 percent. Thus, the summation of the elasticities ($\alpha_1 + \alpha_2, \ldots \alpha'_4$) appears to be < 1.0. We may have diseconomies of scale. The slope or trade-off between ready hours and potential sorties is the lowest (.395) of the five types of aircraft examined. The standard deviation of the slope value (.282) is the largest, indicating a lower confidence in this measure. Again, if flying hours are increased, the slope does increase somewhat, and constant economies of scale might be obtained by such a change in policy.

Variations in the maintenance policy can save about 10 percent of the total labor force (19.8 men). If this \$10.59 K/month wage saving is used to purchase additional spare parts (W_4), we can have an increase of 5.33 sorties or a + 2.04 percent.

Variations in the flying policy may have a sizable effect on output, and variations in the maintenance policy can further increase the output by at least 2.04 percent.

Sensitivity to Cost Changes

For the average A-6 squadron observed, we had the following Cobb-Douglas type relationship:

$$\begin{aligned} U_1 &= 197.01(7)^{.035}(25)^{.050}(20)^{0}(20)^{.19} \\ &\quad + |.395|[11.81(6.6)^{1.075}(173)^{.041}(67)^{.109}(62)^{.152} - 328.2] \\ &= 262.2 \text{ sorties at RH} = 500 \end{aligned} \tag{6.7}$$

With a 10 percent increase in inputs the output increased to 383.7 sorties (increase of 21.5 sorties or 8.21 percent). Specifically the increased sorties were a result of:

+ 15.267 sorties - due to an increase of 1.36 aircraft (W_1)

+ 2.195 sorties - due to an increase of 19.8 men (W_2)

+ 1.430 sorties - due to an increase of 8.7 units support (W_3)

+ 2.648 sorties - due to an increase of 8.2 units spares (W_4)

Comparing the MPP's to their costs will indicate which factors of production should be substituted for another to achieve or approach the lowest cost combination for a fixed output. If the relative ranking changes as the prices are varied for each input, then the direction of substitution will shift, revealing that the priority of substitution is sensitive to price shifts.

Specifically, the range of A-6 MPP/Cost, for each input, is:

TABLE IV

Inputs	$_1C$	$_2C$	$_3C$	$_4C$	Priority of substitution
Aircraft W_1	.176	.162	.108	.212	2nd, 3rd, or 4th
Men W_2	.207	.171	.115	--	2nd, 3rd, or 4th
Support W_3	.223	.175	--	--	2nd or 3rd
Spares W_4	.504	.315	--	--	1st

One input, spare parts (W_4), always surpasses the others, and its relative priority is insensitive to cost change. The other three inputs are sensitive to modest price changes; therefore, for the range of cost under consideration, there is some uncertainty concerning which factors should be substituted to obtain the least cost combination.

Average Cost versus Marginal Cost

Recall that the average observed A-6 squadron production function (6.7) had an output of 262.2 sorties at a sortie policy of RH = 500.

If we hold both the budget ($1,163.7 K/month) and the policy (RH = 500) constant, subject to the following constraints,

$$9 \le W_1 \le 14$$
$$170 \le W_2 \le 220$$
$$70 \le W_3 \le 125$$
$$70 \le W_4 \le 150,$$

and if we maximize the objective function, using basic cost inputs, we have

$$\begin{aligned} U_1' &= 197.01\,(6.4)^{.035}(27)^{.050}(28.7)^{0}\,(36.6)^{.019} \\ &+ |.395|\,[11.81(6.0)^{1.075}(193)^{.041}(96.3)^{.109}(113.4)^{.152} - 328.2] \qquad (6.8) \\ &= 269.97 \text{ or an increase of } 7.77 \text{ sorties or } +2.96 \text{ percent.} \end{aligned}$$

This "best" mix was obtained with the following shifts in inputs:

- 1.20 aircraft

+ 22.00 men

+ 38 units support

+ 68 units spare parts

This "best" mix had an average cost (AC) per sortie of 4.310. But the basic marginal cost (MC) to generate additional sorties by use of each input is the reciprocal of the MPP_{W_j}/C_j, which is:

$$MC_1 = 6.173$$
$$MC_2 = 4.831$$
$$MC_3 = 4.484$$
$$MC_4 = 1.984$$

In this case only one MC (W_4) is less than the AC, and thus a logical candidate to be increased in order to reduce average cost. Unfortunately this factor of production is restricted already by its upper constraint ($W_4 \le 150$). No additional spare parts can be added, as the problem has been defined, and therefore the AC of $4.310K is the lowest possible cost under the specified constraints.

E-2 RESULTS, GENERAL COMMENTS

The regression analysis of the 30 observations again gave about the same search pattern as the values converged to a "best" solution. Specifically, we have:

	α_1	α_2	α_3	α_4	
Sorties (U_1)	.092	--	.051	.044	Σ .187
Ready hours (U_2)	1.005	.149	--	.146	Σ1.300

The coefficient of determination (R^2) was .939. Analysis of the residuals indicated a zero sampling error of central tendency. The proportion of variance error was .347, and the proportion of covariance error was .653. The Durbin-Watson statistic for the observations (when ranked in order of occurrence within a cruise) was 1.495. In this case we have a lack of positive serial correlation but at a level that is not statistically significant. It appears that the time series correlation between the E-2 observations is at least different in magnitude and direction from the other types of aircraft. The E-2 squadrons are much smaller (4 aircraft) than the other squadrons, and they often have external assistance early in the cruise from the E-2 headquarters at Norfolk or San Diego. This outside assistance drops off about mid-cruise. Thus, it is logical that the E-2 learning curve may appear to decrease in the latter half of the cruise, relative to the learning curves of the other squadrons.

The labor elasticity tended to be negative for the sortie output (U_1) but not to a statistically significant degree. Again there is the measurement problem associated with support equipment actually used which may explain the negative tendency for the α_3' elasticity.

<u>Variations in Policy</u>

The level of readiness for the average observations was 59.7 percent at RH = 430. At this level, a 10 percent increase in inputs resulted in an increase in output of 9.80 percent or 6.16 sorties. The summation of the elasticities ($\alpha_1 + \alpha_2, \ldots, \alpha_4'$) is $\approx$ 1.0, so we have near constant returns to scale. Unfortunately, few observations occur above and below the average flying policy level (RH = 430). It is difficult to predict, with high confidence, how the "slope" or trade-offs between ready hours and sorties will shift with a change in the length of the flying day. A scatter diagram of the few points at RH $\geq$ 500 indicates, probably, that we have some economies of scale associated with increasing the length of the flying day.

Again variations in the maintenance policy can save up to 10 percent on total labor (5.5 men). If this $3.201 K/month wage saving is used to purchase additional spare parts (W_4) we can have, at least, an increase of .781 sorties or a 1.27 percent increase in output. A change in the maintenance policy will affect readiness also, and this in turn has the potential of generating additional sorties.

Sensitivity to Cost Changes

In the average E-2 squadron we had the following Cobb-Douglas type production function:

$$
\begin{aligned}
U_1 &= 50.449(.65)^{.092}(20)^{0}(6)^{.051}(30)^{.044} \\
&\quad + |.656| \left[5.2049(3.75)^{1.005}(38)^{.149}(80)^{0}(50)^{.146} - 59.086\right] \qquad (6.9) \\
&= 61.7 \text{ sorties}
\end{aligned}
$$

With a 10 percent increase in inputs, the output increased to 67.859 sorties (increase of 6.159 or 9.98 percent). These increased sorties were a result of:

+ 4.452 sorties - due to an increase of .44 aircraft (W_1)

+ .581 sorties - due to an increase of 5.5 men (W_2)

+ .302 sorties - due to an increase of 8.6 units support (W_3)

+ .824 sorties - due to an increase of 8.0 units spares (W_4).

Comparing the MPP's to their costs will indicate which factors of production should be substituted for each other. The comparison of the MPP's/Cost indices will indicate also if these relative values are sensitive to variations in cost.

Specifically, the range of MPP/Cost, for each input, is:

TABLE V

Input	$_1C$	$_2C$	$_3C$	$_4C$	Priority of substitution
Aircraft W_1	.088	.082	.054	.107	2nd or 3rd
Men W_2	.182	.152	.098	--	2nd or 3rd
Support W_3	.024	.018	--	--	4th
Spares W_4	.244	.094	--	--	1st, 2nd or 3rd

Only one factor of production is insensitive to variations in cost: the support equipment (W_3), which always ranks last. Spare parts (W_4) tend to surpass the other inputs, but W_4 is sensitive to cost variations. On the margin, men (W_2) tend to surpass aircraft (W_1), but this relative ranking can change depending on the price of individual inputs.

Average versus Marginal Cost

Recall that the average observed E-2 squadron production function (6.9) had an output of 61.7 sorties at a sortie policy of RH = 430.

If we hold both the budget (\$655.42 K/month) and the policy (RH = 430) constant, subject to the following constraints,

$$
\begin{aligned}
3.0 &\le W_1 \le 5.0 \\
50 &\le W_2 \le 65 \\
70 &\le W_3 \le 125 \\
70 &\le W_4 \le 150,
\end{aligned}
$$

and if we maximize output, using the basic cost inputs, we have:

$$
\begin{aligned}
U_1' &= 50.449(.65)^{.092}(23)^{0}(5)^{.051}(56)^{.044} \\
&\quad + |.656|[5.205(3.65)^{1.005}(42)^{.149}(65)^{0}(94)^{.146} - 59.086] \\
&= 66.58 \text{ sorties or an increase of 4.88 sorties or 7.91 percent.}
\end{aligned} \tag{6.10}
$$

This "best" mix was obtained with the following shifts in inputs:

- .1 aircraft

+ 10.0 men

- 16.0 units of support

+ 70.0 units of spares

This "best" mix had an average cost (AC) per sortie of \$9.844 K. But the basic marginal cost (MC) to generate additional sorties, by use of each input, is the reciprocal of the MPP_{W_j}/C_j, which is:

$$
\begin{aligned}
MC_1 &= 12.1951 \\
MC_2 &= 5.4545 \\
MC_3 &= 41.6666 \\
MC_4 &= 4.0984
\end{aligned}
$$

In this case, two MCs < AC (W_2 and W_4), but we have already added as many of these units as the constraints will allow ($W_2 \le 65$, $W_4 \le 150$). Therefore, the AC \$9.844 K is the lowest possible average cost for this specified problem.

TOTAL OBJECTIVE FUNCTION

Recall that the total objective function (6.2) was of the type:

Maximize

$$\underset{(A\text{-}7)}{\lambda_1\alpha_0 W_1^{\alpha_1}\ldots\alpha_4^{\alpha_4}} + \underset{(A\text{-}6)}{\lambda_2\alpha_0 W_1^{\alpha_1}\ldots W_4^{\alpha_4}} + \ldots \underset{(F\text{-}4)}{\lambda_4\alpha_0 W_1^{\alpha_1}\ldots W_4^{\alpha_4}} ,$$

subject to the total and individual limitations (upper and lower bounds) for each type of input and the budget constraints.

If the primary purpose of these aircraft is the attack mission, then a best airwing mix would appear to be where the following objective function is a maximum:

$$\left.\begin{array}{l}\text{Max}\left[226.22({}_1W_1)^{.040}({}_1W_2)^{0}({}_1W_3)^{.043}({}_1W_4)^{.032}\right. \\ \left. + \left|.646\right|[9.525({}_2W_1)^{.933}({}_2W_2)^{.050}({}_2W_3)^{0}({}_2W_4)^{.381}-302.6]\right] +\end{array}\right\}\text{A-7}$$

$$\left.\begin{array}{l}\left[(.77 \text{ to } 1.08)[197.01({}_1W_1)^{.035}\ldots({}_1W_4)^{.019}\right. \\ \left. + \left|.395\right|[11.81({}_2W_1)^{1.075}\ldots({}_2W_4)^{.152}-328.2]]\right] +\end{array}\right\}\text{A-6}$$

$$\left.\begin{array}{l}\left[(.88 \text{ to } .97)[187.52({}_1W_1)^{.044}\ldots({}_1W_4)^{.025}\right. \\ \left. + \left|.546\right|[3.21({}_2W_1)^{1.09}\ldots({}_2W_4)^{.120}-262.71]]\right]\end{array}\right\}\text{F-4} \tag{6.11}$$

subject to:

$10 \le {}_{A7}W_1 \le 14$

$120 \le {}_{A7}W_2 \le 150$

$70 \le {}_{A7}W_3 \le 125$

$70 \le {}_{A7}W_4 \le 150$

$9 \le {}_{A6}W_1 \le 14$

$170 \le {}_{A6}W_2 \le 220$

$70 \le {}_{A6}W_3 \le 125$

$70 \le {}_{A6}W_4 \le 150$

$9 \le {}_{F4}W_1 \le 14$

$140 \le {}_{F4}W_2 \le 165$

$70 \le {}_{F4}W_3 \le 125$

$70 \le {}_{F4}W_4 \le 150$

$$\underset{A-7}{C_1W_1}+\underset{A-7}{C_2W_2}+\underset{A-7}{C_3W_3}+\underset{A-7}{C_4W_4}+\underset{A-6}{C_1W_1}+\ldots\underset{A-6}{C_4W_4}+\underset{F-4}{C_1W_1}+\ldots\underset{F-4}{C_4W_4} \le \$2,461.95 \text{ K/mo.}$$

However, a heuristic programming solution to (6.11) may not be too practicable, and from the decision maker's viewpoint may fail to highlight correctly the underlying issues. A good model is no guarantee of success. Probably the individual aircraft constraints are not mutually exclusive of each other. For example, some of the A-7 support equipment can service an A-6, or squadron maintenance personnel working in shop spaces can work on more than one type of aircraft, and some spare parts are common to more than one type of aircraft.

Based upon the specific squadron 3M monthly inputs, a "best" suboptimized objective function was determined for each type of aircraft [see (5.2), (6.4), (6.6), (6.8), and (6.10)].

However, it is not certain that (6.11) will expose the best alternative if only the recorded 3M data is used. More knowledge is needed about the degree of overlapping effects between different squadron inputs and the total airwing output. Then too, there is the problem of the measurement of capital input: idle capital is not reported separately in the 3M data. Recall also that the questionnaire data indicated that a small (< 5 percent) decrease in aircraft assets leads to no decrease in output (queueing problems on the flight deck).

But the sub-optimizing model (3.7) is most useful at the squadron level, and the results from this model offer a way to finesse an indication of the direction to move toward a "better" mix of the inputs in the total objective function (6.11).

Recall that the best average cost (AC) and the utility values (λ) for each type of aircraft that can perform attack missions have been estimated. If these costs are normalized to the A-7 and the λ'/cost ratio (λ/cost to $\bar{\lambda}$/cost) is established, we have:

Aircraft	AC	$AC/AC_{[A-7]}$	λ	$\bar{\lambda}$	λ'/cost ratio range
A-7	1.670	1.000	1.00	1.00	1.00
A-6	4.310	2.580	0.77	1.08	.30 to .42
A-4	0.728	0.436	0.19	0.60	.44 to 1.38
F-4	2.287	1.549	0.88	0.97	.57 to .63

The λ'/cost ratio column indicates that the relative worth of an A-7 aircraft appears to exceed the value of A-6's and F-4's for some missions. The A-4 utility/cost ratio has the widest range and greatest uncertainty concerning its relative worth. At present this aircraft is not scheduled for future airwings.

From this it appears that the outputs obtained from the A-6 and the F-4 resources do not compete with the A-7 for certain scenarios. Of course, the F-4's primary mission is as a fighter and the primary mission of the A-6 is all-weather attack. But this λ'/cost scale indicates the limited pay-off obtained from A-6's and F-4's if they are acquired for other than their primary missions. If we desire to maximize day attack outputs, it appears that some F-4 and A-6 assets should be shifted into the A-7 area. If we need more total sorties (at a higher budget level), we should, in general, increase the spare parts (W_4) up to the space limiting factors. This is an attractive alternative for generating additional outputs at a lower average cost.

SUMMARY

The Cobb-Douglas (CD) type model appears to be reasonably specified and explains most of the observed variation. The maximum likelihood estimate used to obtain the CD elasticities does not appear to have any detectable sampling bias. The coefficient of determination (R^2) varied from .884 to .951. Analysis of the residuals indicated the largest proportion of error was of the covariance type. This is probably due mainly to limitations in the accuracy of the 3M data. Four of the aircraft types observed appear to have a positive serial correlation between observations when the observations are arranged in their order of occurrence within a cruise. However, this correlation was statistically significant only in the cases of the F-4 and A-6. In the case of the E-2 the serial correlation tended to be negative.

With the exception of the A-6, the sea-based aircraft production process of generating sorties and ready hours, appears to operate at near constant economies of scale. However, if the flying day is increased (all other inputs held constant), economies of scale probably take place. In a constant budget situation, variations in both the flying hours (up to about 16 hours per day) and maintenance policy can increase the outputs by as much as 14 percent.

The aircraft production function for each type of aircraft can be maximized for a fixed budget and specified constraints. Under these conditions the potential increased output varied from a + 2.65 percent (A-4) to a + 9.6 percent (A-7). The ratio of the $MPP_j/Cost_j$ gives a good indication of which inputs should be substituted for each other. In general, the marginal value of spare parts (W_4) surpassed the other inputs. In the case of the F-4 the marginal value of support equipment was dominant. With the exception of the E-2 data, labor (W_2) (on the margin) had the lower priority. This correlates closely with the work of Sutton, who established the labor elasticities concerned with Naval aircraft had a range from .02 to .06. The relative priority or the value of additional aircraft (W_1) was sensitive to the cost definitions but in general the MPP of aircraft/cost was not competitive with the alternative of obtaining more spare parts.

In all cases the average cost of an aircraft sortie was greater than marginal cost of certain factors of production. If the budget of the specific ship/airwing and space for spare parts/support are not the major constraints, expanding certain inputs is an attractive source of additional outputs.

Finally, the total airwing attack objective function was described. An optimum heuristic solution may not be possible when using today's data, but an indication can be obtained of which direction to move for a better total airwing solution.

FOOTNOTES

[1]Gerhard Tintner, "A Note on the Derivation of Production Functions from Farm Records," Records," Econometrica, Vol. 12 (1944), p. 29.

[2]Marc Nerlove, Estimations and Identification of Cobb-Douglas Production Functions. Amsterdam: North Holland Publishing Co., 1965, pp. 74-75

SECTION VII

CONCLUSIONS

The key points of the research objectives of this study are restated as follows:

(a) Construct and evaluate an aircraft carrier production function model in order to gain insights into the actual input-output process of operating sea-based tactical aircraft.

(b) Determine the best proportion (allocation) of aircraft, men, support equipment, and spare parts to obtain maximum output under various constraints.

(c) Within various constraints, indicate the optimum squadron composition at various levels of cost to achieve maximum outputs for an airwing.

(d) Identify possible unused resources, if any, and the possible opportunity costs associated with not using these resources.

(e) Indicate whether the allocation decisions pertaining to the use of inputs are sensitive to moderate changes in the price of the factors of production.

(f) Identify the relative "costs" between various combinations of sortie and maintenance policies.

PRODUCTION FUNCTION MODEL

The model appears to fit and explain the variation in the data quite well. One must use caution in evaluating on the basis of a high coefficient of determination. Enough variables can always result in a high R^2 but not necessarily increase the precision of the estimate. In this case, the number of observations is much higher than the numbers of independent variables. The F-test indicates that each variable used contributed to the precision and fit. While the F-statistic is more appropriate as a test of linear models, it can be used as a measure of comparison when we have a non-linear situation. Analysis of the residuals fails to detect any sampling bias. The largest proportion of residual error was of a random covariance type, probably due to the lack of precision and operator error, or both, in the reporting and recording of the basic 3M maintenance information. In addition to the above 3M precision errors, we have some measuring bias, such as the possible distortion associated with the 58 units of support equipment recorded as a measure of the total support population. Fortunately, the marginal costs between alternatives (W_1, W_2, W_3, W_4) are sufficiently robust that the likelihood of incorrectly ranking the management alternatives due to measuring bias is small.

The net sum of the above leads to the tentative conclusion that the production function model fits all of the tested data very closely. The model is consistent with the research of others; all of the independent and dependent variables used appear logical; and these variables are supported by empirical evidence which reflects real world experience. Predictions based upon this model therefore merit serious consideration.

RESOURCE ALLOCATION

In spite of the potential limitations due to the measuring bias noted above and the problem of recording capital services available rather than capital services used, the C-D function continues to show great power. This function permits the decision maker to gain insight into which factors of production should be substituted for each other through the general least-cost substitution principal. The C-D model shows how and when changes in factor prices induce further substitution and how the average cost (AC) compares to the marginal cost (MC) for various capacity restrictions and levels of policy.

The maximizing of the C-D objective function for various levels of the budget clearly indicates that a better allocation of inputs - aircraft, men, support, and spares - is possible at the squadron level. Compared to the Navy's current logistic review process, this methodology offers promise of rapidly arriving at a "best" allocation arrangement for new types of aircraft. For some types of aircraft it appears that a reallocation of items can lead to about a 10 percent increase in output for the same total budget. Since the marginal cost for some factors, usually spare parts, is definitely below average cost, it seems logical to conclude that these relatively low marginal cost inputs are under-utilized. Here are resources that are not being fully taxed. If additional output is needed, increased capital investments should be made to take advantage of these opportunities.

Caution must be taken in expanding the low MC resources. Field trips and secondary information indicate that there are other alternatives that will increase the effective spare parts and support equipment other than the brute force one of buying more. Improved resupply time, selective stocking in depth for high turnover items, increased spare parts for support equipment, and the increased quality of personnel to handle both supply and support equipment are all alternatives to just increasing the input of spare parts (W_4) or support (W_3). The scope of this study was the development of a methodology for identifying the priorities between inputs, not the specifics for implementing the actual procurement actions.

The production function Cobb-Douglas type model evaluated appears to be most appropriate as a management aid to indicate the direction that reallocation should move in order to achieve a "better" total airwing mix between squadrons. The complexity of the airwing problem is of a higher order than that of the squadron suboptimizing issues. If utility values (λ) can be developed for common missions using different types of aircraft, and this study shows a way to establish the bounds of this issue, then the best average cost (AC) per sortie per type of aircraft can also be determined from the squadron objective function. From these two scales (λ and AC) a relative worth (λ/AC) index or scale can be tabulated. For the illustrated example, using modern attack aircraft, this relative worth scale appears to be quite robust and therefore should overcome any subjective limitations associated with a utility scale. At least this methodology can illuminate the underlying issues in order that allocation decisions, at both the airwing and squadron levels, can be made with diminished uncertainty. In the example traced through, it appears that the A-7 squadrons should be both larger and have a greater investment in spare parts, at the expense of other airwing resources for typical attack missions.

The relative values of the various sortie and maintenance policies were difficult to quantify using existing 3M data. Of the feasible maintenance policies considered, a saving of up to about 10 percent in manpower requirements appears possible, along with a

resulting increase in readiness of up to 5 percent. This saving in labor can be used to purchase factors of production that have a higher marginal physical product/cost ratio than labor, with the result that changes in maintenance policies can lead to an equal cost situation with an additional 2-5 percent increase in sortie output. In the sortie policy area, the one alternative that appears sensitive to change is the length of flying day, with the accompanying specification of a lower level in readiness. If the flying day is increased, say up to 16 hours per day, economies of scale in the neighborhood or 10 percent or more appear achievable. This is particularly true in the case of the A-6 aircraft.

There are, of course, transaction costs associated with lengthening the flying day that occur above the airwing level. The costs of extra catapult crews, tower personnel, conflict with replenishment plans, etc., may offset the potential gains. However, if the Navy continues with the new CV concept - a combined attack/fighter and ASW airwing - the transaction cost of having the A-6 squadron fly a 16-hour day might be small: the equal cost output payoff appears to be about a +15 percent. Of course, in considering new policies there is more to the problem than just the logic of the input items considered. The decision maker's experience and attitudes toward changes of this type invariably hinge on far more factors than can be included in this model. At least the Cobb-Douglas type model gives management another tool to expose the underlying problem of allocating scarce resources.

SUMMARY

The C-D type model, in spite of some limitations, shows great power in demonstrating empirical results. It appears reasonably specified and certainly explains most of the observed variation. The model can quantify the marginal pay-off associated with the various inputs. While some doubt may exist concerning the ability of the model to indicate the optimum allocation point, particularly if the probable optimum area requires an extrapolation of existing 3M data, it is quite apparent that this methodology indicates the direction to move toward a better solution. If the decision maker moves in incremental steps toward the optimum area, and updates his model with new observed data, the likelihood of overcorrecting is small and the probability of an increased output per dollar almost certain.

The potential practical applications of this methodology are multifold. It is a method to increase potential output for a fixed budget, a way of exposing under-utilized resources, or a way to model the implications of price changes for the factors of production (i.e., all-volunteer armed forces pay rates). This approach is equally applicable to the airline industry and to military air operations - both desire to maximize the potential output within their capacity restrictions.

FURTHER RESEARCH

Finally, this study appears to invite further research. Further investigation would assist in resolving possible uncertainty pertaining to the full implications of this methodology. Fruitful areas include, but are not limited to, the following:

(a) The possibility exists that the tentative conclusions of this study could be considered inconclusive due to such factors as sampling error, biases in the recorded data, and restrictive assumptions. When records are available that demonstrate an improved collection and measurement of capital, this dilemma may be resolved.

(b) Additional inputs into the Cobb-Doublas type production function could further limit the possibility of misspecification bias. If future maintenance records are specific enough, candidates for inclusion are:

1. The management factor

2. Spare parts for support equipment

3. Subdivisions of labor - direct, indirect labor, skilled, semi-skilled plus civilian or contractor labor aboard ship

4. Power (electrical, compressed air, gasoline) as a substitute for labor

5. Shop space in square feet.

(c) The Durbin-Watson statistic indicated no measurable technical progress in production efficiency between July 1968 to October 1970. Experience with other industrial processes casts doubt on this finding. An expanded sample over a longer time period should resolve this issue.

(d) As mentioned previously in this study, the research effort was limited to Navy 3M information with supporting questionnaires and field trips. It would appear that similar additional research, using U.S. Air Force, Marine Corps, and airline maintenance data would contribute to a better understanding of the input-output relationship in this field.

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APPENDIX A

APPENDIX A

TOTAL POPULATION OF OBSERVATIONS

This appendix lists by actual airplane squadron and ship the total population of observations used in this study. Section IV describes in detail the examples that were drawn from this population.

		JUL	AUG	SEP	OCT	NOV	DEC	JAN	FEB	MAR	APR	MAY	JUN	JUL	AUG	SEP	OCT	NOV	DEC	JAN	FEB	MAR	APR	MAY	JUN	JUL	AUG	SEP	OCT
		1968						1969												1970									
VF-11	AA1					X	X	X	X	X	X													X	X	X			
VF-14	AA2											X	X	X	X	X	X	X	X										
VF-31	AB1														X	X	X	X	X							X	X	X	X
VF-32	AB2											X	X	X	X	X	X	X	X										
VF-33	AD1												X	X	X	X	X	X											
VF-41	AE1																							X	X	X			
VF-74	AF1					X	X	X	X	X	X													X	X	X			
VF-84	AE2																							X	X	X			
VF-102	AD2												X	X	X	X	X	X											
VF-103	AF2														X	X	X	X	X							X	X	X	X
VF-21	PA1					X	X	X	X	X	X	X												X	X				
VF-92	PC1							X	X	X	X	X	X	X											X	X	X	X	X
VF-96	PC2							X	X	X	X	X	X	X											X	X	X	X	X
VF-114	PD2							X	X	X	X	X	X	X	X														
VF-142	PF1	X	X	X	X	X	X	X								X	X	X	X										
VF-143	PF2	X	X	X	X	X	X	X								X	X	X	X										
VF-151	PG1		X	X	X	X	X	X	X	X							X	X	X					X	X				
VF-154	PG2					X	X	X	X	X	X	X												X	X				
VF-161	PH1		X	X	X	X	X	X	X	X							X	X	X					X	X				
VF-213	PK2							X	X	X	X	X	X	X	X														

FIG. A-1: F-4 OBSERVATIONS

		JUL	AUG	SEP	OCT	NOV	DEC	JAN	FEB	MAR	APR	MAY	JUN	JUL	AUG	SEP	OCT	NOV	DEC	JAN	FEB	MAR	APR	MAY	JUN	JUL	AUG	SEP	OCT
		1968						1969												1970									
VA-37	AH3							X	X	X	X	X	X	X	X											X	X	X	X
VA-46	AG3														X	X	X	X	X										
VA-82	AE6																X	X	X					X	X				
VA-86	AE5																X	X	X					X	X				
VA-105	AB6							X	X	X	X	X	X	X	X											X	X	X	X
VA-27	PF7	X	X	X	X	X	X	X								X	X	X	X										
VA-56	PB5																							X	X				
VA-93	PC3																							X	X				
VA-97	PF6	X	X	X	X	X	X	X								X	X	X	X										
VA-113	PD4														X	X	X	X	X										
VA-146	PF5							X	X	X	X	X	X	X											X	X	X	X	X
VA-147	PA7					X	X	X	X	X	X	X													X	X	X	X	X
VA-215	PK4							X	X	X	X	X	X	X										X	X	X			

FIG. A-2: A-7 OBSERVATIONS

		1968						1969												1970									
		JUL	AUG	SEP	OCT	NOV	DEC	JAN	FEB	MAR	APR	MAY	JUN	JUL	AUG	SEP	OCT	NOV	DEC	JAN	FEB	MAR	APR	MAY	JUN	JUL	AUG	SEP	OCT
VA-15	AA4					X	X	X	X	X	X																		
VA-34	AB3					X	X	X	X	X	X																		
VA-36	AB5																							X	X	X			
VA-64	AD3												X	X	X	X	X	X											
VA-66	AD5																							X	X	X			
VA-81	AF3											X	X	X	X	X	X	X	X										
VA-83	AF4											X	X	X	X	X	X	X	X										
VA-106	AG4												X	X	X	X	X	X											
VA-95	PC5											X	X	X	X	X	X	X	X										
VA-153	PG4		X	X	X	X	X	X	X	X																			
VA-155	PG5					X	X	X	X	X	X	X																	
VA-216	PK5		X	X	X	X	X	X	X	X														X	X	X			
VSF-1	QAO												X	X	X	X	X	X											

FIG. A-3: A-4 OBSERVATIONS

		JUL	AUG	SEP	OCT	NOV	DEC	JAN	FEB	MAR	APR	MAY	JUN	JUL	AUG	SEP	OCT	NOV	DEC	JAN	FEB	MAR	APR	MAY	JUN	JUL	AUG	SEP	OCT
		1968						1969												1970									
VA-35	AB4																X	X	X					X	X				
VA-65	AD4							X	X	X	X	X	X	X	X														
VA-75	AE4														X	X	X	X	X							X	X	X	X
VA-85	AF5															X	X	X	X										
VA-176	AG5																							X	X	X			
VA-52	PB3		X	X	X	X	X	X	X	X																			
VA-145	PF4							X	X	X	X	X	X	X	X														
VA-165	PH5					X	X	X	X	X	X	X													X	X	X	X	X
VA-196	PJ5	X	X	X	X	X	X	X																X	X				

FIG. A-4: A-6 OBSERVATIONS

		1968						1969												1970									
		JUL	AUG	SEP	OCT	NOV	DEC	JAN	FEB	MAR	APR	MAY	JUN	JUL	AUG	SEP	OCT	NOV	DEC	JAN	FEB	MAR	APR	MAY	JUN	JUL	AUG	SEP	OCT
VAW-112	PR2							X	X	X	X	X	X	X															
VAW-113	PR3	X	X	X	X	X	X	X								X	X	X	X										
VAW-114	PR4							X	X	X	X	X	X	X	X														
VAW-115	PR5					X	X	X	X	X	X	X												X	X				
VAW-116	PR6		X	X	X	X	X	X	X	X							X	X	X					X	X				
VAW-122	AJ1												X	X	X	X	X	X											
VAW-124	AJ4																								X	X	X	X	X
VAW-125	AJ5														X	X	X	X	X										
VAW-126	AJ6																							X	X	X			

FIG. A-5: E-2A OBSERVATIONS

		1968						1969												1970									
		JUL	AUG	SEP	OCT	NOV	DEC	JAN	FEB	MAR	APR	MAY	JUN	JUL	AUG	SEP	OCT	NOV	DEC	JAN	FEB	MAR	APR	MAY	JUN	JUL	AUG	SEP	OCT
J.F.K.	CVA-67											X	X	X	X	X	X	X	X										
F.D.R.	CVA-42																							X	X	X			
Forrestal	CVA-59					X	X	X	X	X	X													X	X	X			
Saratoga	CVA-60														X	X	X	X	X							X	X	X	X
Independence	CVA-62												X	X	X	X	X	X											
America	CVA-66																								X	X	X	X	X
Coral Sea	CVA-43	X	X	X	X	X	X	X	X	X							X	X	X					X	X				
Kitty Hawk	CVA-63							X	X	X	X	X	X	X	X														
Enterprise	CVA-65							X	X	X	X	X	X	X															
Ranger	CVA-61					X	X	X	X	X	X	X												X	X				
Constellation	CVA-64	X	X	X	X	X	X	X								X	X	X	X										

FIG. A-6: CVA OBSERVATIONS

APPENDIX B

APPENDIX B

READY HOURS VERSUS SORTIES

This figure shows the actual relationships between observed sorties versus ready hours for the F-4 aircraft as described in section IV. This represents a total population, less first, last, December and adjacent months observations and inputs from the aircraft carrier J. F. Kennedy.

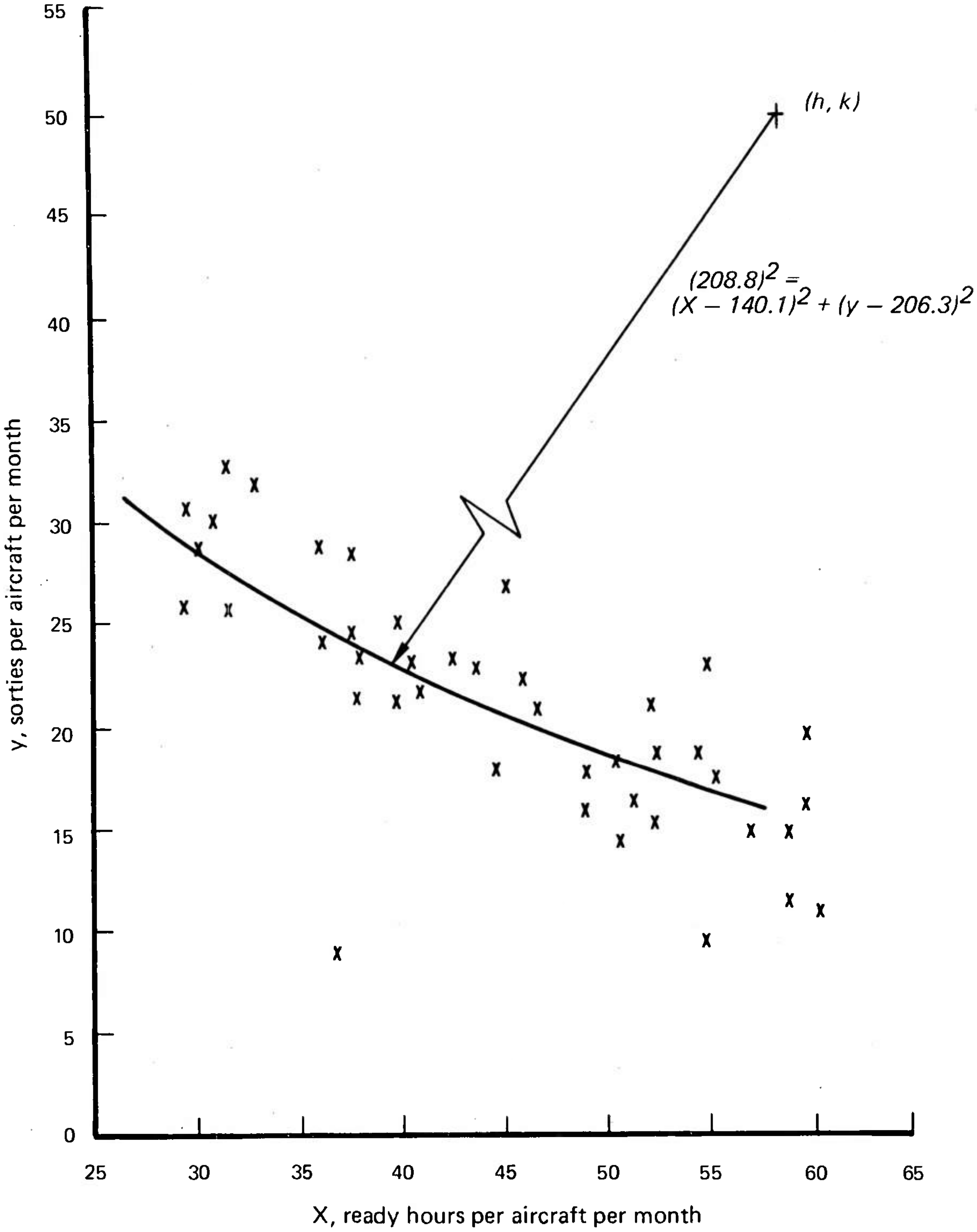

FIG. B-1: READY HOURS VERSUS SORTIES

APPENDIX C

APPENDIX C

QUESTIONNAIRE

This is a copy of the questionnaire sent out to eleven Air Department officers of U.S. aircraft carriers. A summary of the eight replies is in section IV.

Center for Naval Analyses

1401 Wilson Boulevard Arlington, Virginia 22209 703/524-9400

an affiliate of the University of Rochester

MEMORANDUM FOR AIR DEPARTMENT USS ____________________

From: Project Officer, Air Wing Composition Study

Subj: Inputs for the Center for Naval Analyses' Sponsored Air Wing Composition Study

Encl: (1) Aircraft Carrier Questionnaire

1. Background. The Center for Naval Analyses is sponsoring the synthesizing of a composite model to handle Air Wing Composition trade off questions. Part of the problem is to determine the relationship, if any, between aircraft ready hours/sorties and resource inputs such as number of aircraft, maintenance manpower, spare parts, and support equipment. The staffs of CNAP and CNAL and the Maintenance Support Office at Mechanicsburg are supplying us with the required specific data.

2. To assist in this effort, request your informal opinion of certain deck loads, support equipment, etc., type inputs as factors to generate sorties. Enclosure (1) indicates the type of information desired.

Respectfully,

CHANTEE LEWIS
Captain USN

ENCLOSURE 1

AIRCRAFT CARRIER COMPOSITION STUDY

QUESTIONNAIRE

1. Name of carrier ______________________ date ______________________.

2. Fleet Commanders/Type Commander's deck multiple ______________________
A-4, A-7 equivalents (strike out one).

3. Estimate average deck multiple for past 30 days ______________________.

4. Assume you are flying 12 hours out of each 24 hours, under combat conditions (full bomb-loads, ECM equipment, airborne CAP, etc.) and the problem of spare parts and support equipment is not a limiting factor. With your present deck multiple, what is your estimated number of sorties you can generate out each day, at a sustained rate? ______________

5. Everything is the same as described in question 4 above, except the deck multiples have been increased by 10%. How would this change your estimate of the number of daily sorties that could be sustained?

Increase 10%	__________
Increase 5%	__________
No change	__________
Decrease 5%	__________
Other (specify)	__________

6. Everything is the same as described in question 4, except the deck multiple has been decreased by 5%. How would this change your estimate of the number of daily sorties possible?

Decrease 5%	__________
No change	__________
Increase 5%	__________
Other (specify)	__________

7. With your present deck multiple, present NORS rate, and at times problems associated with support equipment, etc. what is your regular daily sortie rate (12 hours flying, 12 hours off)? ______________

8. If Supply could increase all on-board aviation spares by 20%, how do you estimate this would effect your sortie rate?

15% increase	__________
10% increase	__________
No change	__________
Other (specify)	__________

Aircraft Carrier Composition Study Questionnaire (Continued)

9. If the AMMRL (Aircraft Maintenance Material Readiness List) and IMRL (Individual Material Readiness List) equipment were increased by 10% (and this new equipment was all operating), how do you estimate this would effect your sortie rate?

   10% increase __________

   5% increase __________

   No change __________

   Other (specify) __________

10. If quality aircraft maintenance personnel were increased by 10% (and you have living, messing spaces for them), how do you estimate this would effect your sortie rate?

    10% increase __________

    5% increase __________

    No change __________

    Other (specify) __________

11. If the ship had an <u>extra</u> one million dollars to spend only on items that would increase the potential sortie rates on what would you spend it and why?____________________

    ____

    ____

    ____

Thank you for your cooperation. Using the self addressed envelope, please return this questionnaire to:

Capt. C. Lewis USN
Center for Naval Analyses
1401 Wilson Boulevard
Arlington, Virginia 22209

SUPPLEMENT TO QUESTIONNAIRE

1. When components are taken out of aircraft and sent to the ship's IMA for repair, sometimes the ship finds the component in question has "no defects". This causes a loss in maintenance man hours sometimes due to limited line support equipment, improper pilot write up, etc. On the average when a "no defect" item is removed, replaced and checked, how much total man hours are involved per "no defect" action, by type of aircraft?

Aircraft	Hours per actions
A-4	________
A-6	________
A-7	________
F-4	________
E-2	________

(leave blank if unknown or or type of aircraft not on board)

2. Sometimes items are sent to IMA that are later determined to be beyond the capability of maintenance (BCM) due to lack of facilities (Code "2") etc. On the average how much time is spent per BCM item (in hours) before this determination is made?

____

SUPPLEMENT TO QUESTIONNAIRE FOR CERTAIN AVCAL ESTIMATES

1. For the time periods indicated below, request your estimates of AVCAL percentages aboard your ship.

Time Period	Percentage
______________________	________
______________________	________
______________________	________
______________________	________
______________________	________
______________________	________
______________________	________
______________________	________
______________________	________
______________________	________

2. If you have an approximate estimate of the dollar value of a full (100%) AVCAL list and the volume (ft^3), that it takes up aboard your aircraft carrier, please indicate.

Value ________________

Volume ______________

APPENDIX D

APPENDIX D

THE CDC 3800 COMPUTER PROGRAM OF THE PRODUCTION FUNCTION MODEL

Section V gives a brief description with block diagram (figure 4) of this program.

FTN5.5A

```
      PROGRAM ACPP MODL
      DIMENSION SP(2,3),U(2,99),W(4,99),W1(99,5),W2(99,5),U1(99),U2(99),
     *P(4),Q(4),A(5),AP(5),Z1(4,99),Z2(4,99),WD(4,99),IV(3,99),ITAG(4),
     *WS(22),WW1(99,5),WW2(99,5),NOS(6,500),AB(4,500),R(4,500),S(4,500)
      DIMENSION GRAD(3)
      DATA (GRAD=.1,.01,.001)
      DIMENSION XD(8,99),D(5),DP(5)
      DIMENSION XV(8,99,11)
      DATA (SP=.5,.5,.75,.25,.25,.75)
      READ 300,NOOB,NOIT
300   FORMAT(I5,35XI5)
      NOB=NOOB*3
      IN=0  $  READ 31,(IV(I),I=1,NOB)
31    FORMAT (9A8)
30    READ (1) ITAG,WS
      IF (EOF,1)   34,35
35    DO 32 I=1,NOOB
32    IF (ITAG(2).EQ.IV(1,I).AND.ITAG(3).EQ.IV(2,I).AND.ITAG(4).EQ.IV(3,
     *I))          GO TO  33
      GO TO 30
33    IN=IN+1
      U(1,IN)=WS( 8)
      U(2,IN)=WS(10)
      W(1,IN)=WS( 2)
      W(2,IN)=WS(20)
      W(3,IN)=WS(21)
      W(4,IN)=WS(22)
      IF (IN.EQ.NOOB)      36,30
34    PRINT 38,IN   $   STOP
38    FORMAT(*0NO OF OBSERVATIONS NE NOOB. (*,I3,* ) JOB TERMINATED.*)
36    XMIN=10**14
      DO 1 M=1,3
      N=1
      XXM=NM=10**14
      DO 2 I=1,NOOB
      U1(I)=LOGF(U(1,I))      $      U2(I)=LOGF(U(2,I))
      W1(I,1)=W2(I,1)=1.0
      DO 2 J=2,5
      W1(I,J)=      SP(1,M)*W(J-1,I)
2     W2(I,J)=      W(J-1,I)-W1(I,J)
13    DO 40 I=1,NOOB
      WW1(I,1)=W1(I,1)  $  WW2(I,1)=W2(I,1)  $  DO 40 J=2,5
      WW1(I,J)=LOGF(W1(I,J))
40    WW2(I,J)=LOGF(W2(I,J))
      CALL REGRESS (U1,WW1,A,NOOB)
      CALL REGRESS (U2,WW2,AP,NOOB)
      IF (N.EQ.1)  PRINT 100
100   FORMAT(*1*,9X,*N*,7X,*1WI*,7X,*2WI*,8X,*A0*,8X,*A1*,8X,*A2*,8X,*A3
     **,8X,*A4*,7X,*AP0*,7X,*AP1*,7X,*AP2*,7X,*AP3*,7X,*AP4*/)
      PRINT 101,N,SP(1,M),SP(2,M),A,AP
101   FORMAT(XI10,2F10.2,10F10.4)
      DO 3 I=1,4
3     P(I)=Q(I)=R(I,N)=S(I,N)=NOS(I,N)=0  $  NOS(5,N)=NOS(6,N)=0
      DO 4 I=1,NOOB  $  DO 4 J=1,4
      Z1(J,I)=U1(I)-A(1) $ Z2(J,I)=U2(I)-AP(1)
      DO 5 K=2,5
      IF (K.NE.J+1)   GO TO 66
      Z1(J,I)=Z1(J,I)-A(K)*LOGF(W(J,I))
      Z2(J,I)=Z2(J,I)-AP(K)*LOGF(W(J,I))     $     GO TO 5
```

```
66    Z1(J,I)=Z1(J,I)-A(K)*LOGF(W1(I,K))
      Z2(J,I)=Z2(J,I)-AP(K)*LOGF(W2(I,K))
5     CONTINUE
      R(J,N)=R(J,N)   +(Z1(J,I)-A(J+1)*LOGF(W1(I,J+1)/W(J,I)))**2
      S(J,N)=S(J,N)   +(Z2(J,I)-AP(J+1)*LOGF(1-(W1(I,J+1)/W(J,I))))**2
      P(J)=P(J)+(Z1(J,I)-A(J+1)*(W1(I,J+1)/W(J,I)-1))**2
4     Q(J)=Q(J)+(Z2(J,I)+AP(J+1)*W1(I,J+1)/W(J,I))**2
      DO 250 J=1,4
250   AB(J,N)=LOGF(R(J,N))+LOGF(S(J,N))
      IS1=IS2=1
      DO 6 I=1,NOOB  $  DO 6 J=1,4
      AA=P(J)*AP(J+1)**2-Q(J)*A(J+1)**2
      BB=2*Q(J)*A(J+1)**2+Q(J)*A(J+1)*Z1(J,I)+P(J)*AP(J+1)*Z2(J,I)
      CC=-Q(J)*A(J+1)*(A(J+1)+Z1(J,I))
      IF ((BB**2-4*AA*CC).LT.0)  GO TO 700
      R1=(-BB+SQRTF(BB**2-4*AA*CC))/2*AA
      R2=(-BB-SQRTF(BB**2-4*AA*CC))/2*AA
      Q1=MIN1F(R1,R2)   $   Q2=MAX1F(R1,R2)
      IF (Q1.LE.0.AND.Q2.LE.0)  GO TO 500
      IF (Q1.GE.1.AND.Q2.GE.1)  GO TO 502
      IF (Q1.LE.0.AND.Q2.GE.1)  GO TO 503
      QQ=Q1   $   IF (Q1.LE.0)   QQ=Q2   $   GO TO 501
700   NOS(1,N)=NOS(1,N)+1   $   GO TO 505
500   NOS(2,N)=NOS(2,N)+1   $   WD(J,I)=.001   $   GO TO 6
502   WD(J,I)=W(J,I)-.001   $   NOS(4,N)=NOS(4,N)+1   $   GO TO 6
503   NOS(5,N)=NOS(5,N)+1
505   WD(J,I)=W1(I,J+1)   $   GO TO 6
501   WD(J,I)=W(J,I)*QQ   $   NOS(3,N)=NOS(3,N)+1
6     IF (ABSF((WD(J,I)-W1(I,J+1))/W1(I,J+1)).GT..01)   IS2=2
      LSW=1   $   IF (      AB(1,N)  .GE.XXM.OR.N.LE.5)   GO TO 704
      LSW=2   $   XXM=      AB(1,N)     $   NM=N
704   DO 12 I=1,NOOB
121   DO 12 J=2,5
      W1(I,J)=WD(J-1,I)   $   W2(I,J)=W(J-1,I)-W1(I,J)  $ GOTO(12,120),LSW
120   XD(J-1,I)=W1(I,J)   $   XD(J+3,I)=W2(I,J)
      D(1)=A(1)  $ DP(1)=AP(1)  $ D(J)=A(J)  $ DP(J)=AP(J)
12    CONTINUE
      IF  ( M .EQ. 1 .AND. N .GE. 10 .AND. N .LE. 20 ) GO TO 730
      GO TO (9,10),IS2
 730  DO 731 I=1,NOOB  $  DO 731 J=1,4   $   XV(J,I,N-9)=W1(I,J+1)
 731  XV(J+4,I,N-9)=W2(I,J+1)
      GO TO (9,10),IS2
10    N=N+1  $   IF (N.GT.NOIT)   11,13
  11  PRINT 101,N
9     PRINT 105,(NM,I,(XD(J,I),J=1,8),I=1,NOOB)
105   FORMAT(*1*,4X,*N*,
     *            4X,*I*,12X,*1W1*,12X,*1W2*,12X,*1W3*,12X,*1W4*,12X,*2W1
     **,12X,*2W2*,12X,*2W3*,12X,*2W4*//(X,2I5,8(7XF8.3)))
      IF (M.NE.1)  GO TO 780
      DO 740 K=1,11  $   KK=K+9
 740  PRINT 105,(KK,I,(XV(J,I,K),J=1,8),I=1,NOOB)
780   IF (N.GT.NOIT)  N=NOIT
      PRINT 105,(N,I,(W1(I,J),J=2,5),(W2(I,J),J=2,5),I=1,NOOB)
      PRINT 670,((NOS(J,I),J=1,6),I,(AB(J,I),J=1,4),I=1,NOIT)
 670  FORMAT(*1*,8X,*IM*,9X,*-*,7X,*0-1*,9X,*+*,7X,*-/+*,7X,* NG*,
     *                                                7X,*  N*,4X,*(A
     *+B)1*,4X,*(A+B)2*,4X,*(A+B)3*,4X,*(A+B)4*//(X,7I10,4F10.3))
20    X3=0   $   DO 15 I=1,NOOB
      X1=X2=0.0
      DO 14 J=2,5
      X1=X1+D(J)*LOGF(XD(J-1,I))
```

```
14    X2=X2+DP(J)*LOGF(XD(J+3,I))
15    X3=X3+(U1(I)-D(1)-X1)**2+(U2(I)-DP(1)-X2)**2
      IF (X3.GE.XMIN)  GO TO 1
      XMIN=X3  $  INDEX=M
1     CONTINUE
      PRINT 106,SP(1,INDEX),SP(2,INDEX),XMIN
106   FORMAT(*1  MIN (*,F4.2,*,*,F4.2,* ) **,F10.5)
      END
```

5.4DS ACPPMODL

```
                                              IDENT      ACPPMODL
     PROGRAM LENGTH                    56374
     ENTRY POINTS        ACPPMODL      54722
     EXTERNAL SYMBOLS
                         Q8QENTRY
                         THEND.
                         Q8QSTOPS
                         Q2Q07000
                         Q8QDICT.
                         REGRESS
                         SQRTF
                         MIN1F
                         MAX1F
                         LOGF
                         Q8QIFEOF
                         TSH.
                         TSB.
                         STH.
                         SLO.
                         SLI.
                         QNSINGL.
        00367 SYMBOLS
```

FTN5.5A

```
          SUBROUTINE REGRESS (Y,X,B,NOOB)
          DIMENSION Y(99),X(99,5),B(5),C(5),XTX(5,5),XTY(5)
          DO 8 I=1,5  $  DO 8 J=1,5
     8    XTY(I)=XTX(I,J)=0
          DO 1 I=1,5  $  DO 1 J=1,NOOB
     1    XTY(I)=XTY(I)+Y(J)*X(J,I)
          DO 2 I=1,5  $  DO 2 J=1,5  $  DO 2 K=1,NOOB
     2    XTX(J,I)=XTX(J,I)+X(K,J)*X(K,I)
          CALL INVERT (XTX,5,B,C)
          DO 3 I=1,5  $  B(I)=0.0  $  DO 3 J=1,5
     3    B(I)=B(I)+XTX(I,J)*XTY(J)
          END
```

```
5.4DS   REGRESS

                                              IDENT      REGRESS
        PROGRAM LENGTH                     00343
        ENTRY POINTS      REGRESS          00046
        EXTERNAL SYMBOLS
                          Q8QDICT.
                          INVERT
           00075 SYMBOLS
LOAD
RUN,5,5000

PROGRAM NAMES
1 21403  ACPPMODL  56374     1 21040  REGRESS   00343     1 20604  ALLOC.    00234
1 16304  XFIXF     00032     1 16262  Q8QLOADA  00022     1 16250  Q2QLOADA  00012
1 14443  IOP.      01400     1 13441  INVERT    01002     1 13367  SLI.      00052
1 12555  IOB.      00554     1 12510  TSH.      00045     1 12455  Q8QIFIOC  00033
1 12240  MAX1F     00121     1 12201  SQRTF     00037     1 12077  ITOJ      00102
1 11412  IOS.      00425     1 11334  Q8QENTRY  00056

PROGRAM EXTENS.
NONE

LABELED COMMON
NONE

NUMBERED COMMON
NONE

ENTRY POINTS
0 77777  SENTRY              1 76325  ACPPMODL            1 11340  Q8QENTRY
1 12037  Q8QSTOPS            1 12102  Q2Q07000            1 11334  Q8QDICT.
1 12210  SQRTF               1 12266  MIN1F               1 12243  MAX1F
1 12464  Q8QIFEOF            1 12514  TSH.                1 12560  TSB.
1 13375  SLO.                1 13367  SLI.                1 11723  QNSINGL.
1 14446  IOP.                1 11465  Q8QHIST.            1 16044  Q8QERROR
1 16250  Q2QLOADA            1 16262  Q8QLOADA            1 16313  XFIXF
1 16362  IOH.                1 11432  IOS.                1 11735  IOR.
1 16336  BCDBUF.             1 11726  QNDOUBL.            1 20742  ALLOC.
1 20716  BUSY.               1 20725  IRETURN.            1 21006  ALLOCIN.
1 20436  .REPCNT.            1 16307  Q8QXFIXF            1 16307  Q8QXINTF
1 16303  Q8QLDCON            1 16300  Q8QLODA             1 16043  Q8NOTRAC
1 15467  ETAB.               1 12713  ELB.                1 12572  STB.
1 12364  Q8QLOGF             1 12247  MAXOF               1 12272  MINOF
1 12256  XMAX1F              1 12266  XMINOF              1 12301  XMIN1F
1 12113  ITOJ                1 12107  Q8QITOJ             1 12045  Q8QPAUSE

EXECUTION STARTED AT    1704 -21
```

APPENDIX E

APPENDIX E

COST OF INPUTS (AIRCRAFT, MEN, SUPPORT AND SPARE PARTS) FOR SELECTED TYPES OF AIRCRAFT

A description of these inputs is contained in section II, specifically p.   for aircraft, p.   for men, p.   for support equipment, and p.   for spare parts.

Tables I through V show a tabulated use of these costs.

COST OF INPUTS/MONTH/PER AIRCRAFT[1]
(In dollars)

Inputs	F-4	A-7	A-6	A-4	E-2
	Type aircraft system				
${}_1C_1$	42,625	27,060	63,800	8,800	114,400
${}_2C_1$	46,242	29,353	69,217	11,080	124,107
${}_3C_1$	69,363	44,030	103,825	16,625	186,160
${}_4C_1$	35,083	22,333	52,917	6,333	94,333
Manpower					
${}_1C_2$	466	443	535	419	582
${}_2C_2$	578	555	647	531	694
${}_3C_2$	858	820	968	782	1,073
Support					
${}_1C_3$	666	695	737	299	1,445
${}_2C_3$	786	815	937	379	1,935
Spart parts					
${}_1C_4$	869	751	641	381	422
${}_2C_4$	1,080	905	1,025	476	1,096

[1]Source: CVAN-70 Aircraft Carrier, op. cit., pp. 104-109.

DEFINITION OF COST CATEGORIES[2]

${}_1C_1$	=	Cost of a UE aircraft, less attrition
${}_2C_1$	=	Cost of a basic UE aircraft
${}_3C_1$	=	Cost of a UE aircraft accelerated to 2/3's DoD expected life
${}_4C_1$	=	PV type of cost of a UE aircraft with discounting (10%) for attrition, overhaul and major changes
${}_1C_2$	=	Basic man month labor cost, Navy planning data
${}_2C_2$	=	Basic man month labor cost plus cost for space
${}_3C_2$	=	All volunteer labor cost plus cost for space
${}_1C_3$	=	Cost of a unit of support equipment
${}_2C_3$	=	Cost of a unit of support equipment plus cost for space
${}_1C_4$	=	Cost of a unit of spare parts
${}_2C_4$	=	Cost of a unit of spare parts plus cost of space

[2] Supra., pp. 43-57, for an expanded description of these categories.

Unclassified
Security Classification

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The Use of Simultaneous Equation Models for Decisions Pertaining to the "Best" Mix Between Aircraft, Spare Parts, Support Equipment, and Support Personnel

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13. ABSTRACT

This is a study of the application of production functions to sea-based tactical air resources: aircraft, spare parts, support equipment, and support personnel. The goal is to develop objective criteria for allocating money among these competing demands using sorties or aircraft ready hours as the output.

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aircraft maintenance						
maintenance costs						

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